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Use of input-output analysis in estimating the interdependence of agriculture and other economic sectors

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USE OF INPUT-OUTPUT ANALYSIS IN ESTIMATING THE INTERDEPENDENCE OF AGRICULTURE AND OTHER ECONOMIC SECTORS

by

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I. INTRODUCTION

As long as government continues its role in determining policies which are intended to regulate markets and production in the economy, agricultural economists will want to direct their efforts toward analyzing the effects of such policy decisions. The analysis should indicate the possible effects of current policies as well as assist administrative units to select those policies which will attain the end in view.

Aggregate effects are of interest when policy is designed to deal with whole sectors of the economy. Legislative and executive groups of government are placing more emphasis upon policies which give less interest to commodity groups and more on agriculture as a whole in order to attain broader ends in view. Research designed to give information and guidance to this type of policy making necessitates methodology which deals with the sectors of the economy as an aggregate or interdependent units producing goods and services to satisfy the wants of society. The Leontief input-output analysis seems to be an appropriate statistical tool for these purposes.

Leontief's work first appeared in 1936 (21, pp. 109-132), and a more complete investigation was published in

1951 (22). Recent interests in the Leontief system of interdependence of industry relations is evidenced by the attention of such groups as the Bureau of Labor Statistics, the Air Force, Army and Navy, Bureau of the Budget, Bureau of Mines, Department of Commerce, Rand Corporation, and many colleges and universities in developing techniques and research studies closely related to Leontief's pioneer work (4, p. 97).

Probably the most important contribution of the Leontief formulation to recent developments in research methodology is its relationship to the development of linear programming. It provided one of the sources of ideas of the programming technique pioneered by a small group in the U. S. Air Force and more extensively investigated by the Cowles Commission (20, p. 3).

With this recent interest in the use of the Leontief system, the question arises as to what extent the Leontief analysis is applicable to problems in agricultural production research. Historically, government policies have attempted to control agricultural production through acreage control and marketing quotas during periods of declining farm prices. On the other hand, policy has been directed at increased agricultural production under production goal programs to meet war and defense needs of the economy. During periods when policies advocate a change of agricultural output through control programs, what effect do such policies have upon the output of other sectors of the economy? What quantities of resources will be released or required in the agricultural economy and related sectors because of these new outputs? Smith (26, p. 138) states that Leontief input-output analysis is appropriate for dealing with these problems. The attempt to employ a Leontief model in estimating empirical interdependence coefficients between the agricultural economy and the other sectors of the economy is the overall objective of this investigation.

The specific objectives are as follows:

1. To formulate a mathematical model of input-output analysis of agriculture and other sectors of the economy.

2. To indicate the adaptability of the Leontief input-output analysis to agricultural production research directed toward obtaining information on the interdependence between agriculture and the rest of the economy.

3. To provide an empirical illustration of the use of the Leontief system by estimating the parameters from presently available statistics of agriculture and other sectors of the economy.

4. To analyze an empirical solution of a Leontief system to obtain information on the interdependence of

agriculture and other sectors of the economy.

5. To provide information for improving the empirical model.

6. To observe the changes in input-output relations and interdependence among the sectors over time from input-output analyses of three points in time.

7. To investigate the validity of the theoretical assumptions of the Leontief system when applied to practical problems of the agricultural economy.

8. To propose additional areas of agricultural research where input-output analysis may apply in seeking solutions to economic problems.

It should be emphasized that the empirical section of this investigation is purely a pilot analysis of economic problems with the aid of empirical interdependence coefficients obtained from the solution of the system. The results may have little practical meaning in the light of observed phenomena in the economic environment of the economy. Such experiences should not bring condemnation upon the methodology but should furnish information for future redesigning of the model, improving the methods of aggregation of the data, or furnishing guides for collecting new and more reliable data for estimating the variables in the model. Time and money resources have been limiting factors in this investigation. When one considers the vast resources available to those agencies now employing the Leontief technique where several years are devoted to collecting the basic data, it is not difficult to realize that the empirical results of a simple investigation such as this one must be looked upon with considerable reservation.

Since input-output analysis is related to the more recent methodology of activity analysis, it is hoped that this investigation will provide the student of economics with a basic understanding of input-output analysis before proceeding to the more mathematical subject, linear programming.

II. THE MATHEMATICAL MODEL

The mathematical technique used in this investigation was developed by Wassily W. Leontief, Professor of Economics at Harvard University. The theoretical scheme developed by him in studying a similar problem involving consumption, employment, investment, and output of the American economy was applicable to this investigation (22, pp. 139-163).

The model for the present investigation consists of five equations, since five economic sectors were used in analyzing the interrelationship of agriculture and the rest of the economy. The mathematical notation could be more condensed than the reader finds in this section; however, since this is the first of a series of input-output studies, the writer has chosen the more explicit method of presentation.

The Leontief input-output analysis is basically an open system of linear equations describing the relationship between the flows of net output between each sector of the economy and household consumption.[1] The solution of this

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[1]Empirical data are actually collected for six sectors. Household consumption is not explained by the system therefore no equation appears in the system for the household sector. The system is therefore called an open system of equations (26, p. 133).

system describes the relationship between household consumption and the net output of the various sectors of the economy. These systems of linear equations will be algebraically derived in the description of the mathematical model. The important underlying assumptions of the input-output technique will be discussed in the theoretical framework but not from the standpoint of their validity in an empirical investigation. This has been reserved for a later section.

The data were aggregated into six sectors of the economy: primary agricultural production, secondary agricultural production, all other industries and services, foreign trade, government, and households. The detailed description of the variables in the system is included in a later section. The flows of net output of the first five sectors are represented by x_{12}, x_{13}, x_{14}, x_{15}; x_{21}, x_{23}, x_{24}, x_{25};--------; x_{51}, x_{52}, x_{53}, x_{54}. x_{12} is the amount of the net output of the first sector consumed by the second sector in its production activity during a period of twelve months. The net outputs of each sector are represented by X_1, X_2, X_3, X_4, X_5. Net outputs are defined as:

$$X_i = \sum_{j=1}^{5} x_{ij} + y_i \qquad i \neq j \; (i = 1,2,3,4,5) \qquad (2.0)$$

where X_i is the net output of the i-th sector; x_{ij} is the quantity of net output of the i-th sector consumed by the j-th sector; and y_i is the portion of the net output of the i-th sector included in the final bill of goods.

Each sector of the economy requires the products from other sectors in producing its own net output which in turn is distributed among the other sectors and the final bill of goods. The balance of physical flows in the economy is described in the first set of five equations.

$$x_{12} + x_{13} + x_{14} + x_{15} + y_1 = X_1 \tag{2.1}$$

$$x_{21} + x_{23} + x_{24} + x_{25} + y_2 = X_2 \tag{2.2}$$

$$x_{31} + x_{32} + x_{34} + x_{35} + y_3 = X_3 \tag{2.3}$$

$$x_{41} + x_{42} + x_{43} + x_{45} + y_4 = X_4 \tag{2.4}$$

$$x_{51} + x_{52} + x_{53} + x_{54} + y_5 = X_5 . \tag{2.5}$$

This system of equations can be rewritten in a more convenient form as follows:

$$X_1 - x_{12} - x_{13} - x_{14} - x_{15} = y_1 \tag{2.6}$$

$$-x_{21} + X_2 - x_{23} - x_{24} - x_{25} = y_2 \tag{2.7}$$

$$-x_{31} - x_{32} + X_3 - x_{34} - x_{35} = y_3 \tag{2.8}$$

$$-x_{41} - x_{42} - x_{43} + X_4 - x_{45} = y_4 \tag{2.9}$$

$$-x_{51} - x_{52} - x_{53} - x_{54} + X_5 = y_5 . \tag{2.10}$$

The next step in the development of the mathematical model is based on the assumption of fixed technical production coefficients in all sectors of the economy. The fixed technical production coefficients of the five sectors included in the system of equations are represented by a_{12}, a_{13}, a_{14}, a_{15}; a_{21}, a_{23}, a_{24}, a_{25}; a_{31}, a_{32}, a_{34}, a_{35}; a_{41}, a_{42}, a_{43}, a_{45}; a_{51}, a_{52}, a_{53}, a_{54}. The coefficient a_{12} is the quantity of physical net output of the first sector used by the second sector per unit of net output produced by the second sector.

The technical relationships between the flows of commodities as total inputs to a sector and its corresponding level of net output are given in the next system of equations.

$$x_{12}=a_{12}X_2;\ x_{13}=a_{13}X_3;\ x_{14}=a_{14}X_4;\ x_{15}=a_{15}X_5 \quad (2.11)$$

$$x_{21}=a_{21}X_1;\ x_{23}=a_{23}X_3;\ x_{24}=a_{24}X_4;\ x_{25}=a_{25}X_5 \quad (2.12)$$

$$x_{31}=a_{31}X_1;\ x_{32}=a_{32}X_2;\ x_{34}=a_{34}X_4;\ x_{35}=a_{35}X_5 \quad (2.13)$$

$$x_{41}=a_{41}X_1;\ x_{42}=a_{42}X_2;\ x_{43}=a_{43}X_3;\ x_{45}=a_{45}X_5 \quad (2.14)$$

$$x_{51}=a_{51}X_1;\ x_{52}=a_{52}X_2;\ x_{53}=a_{53}X_3;\ x_{54}=a_{54}X_4; \quad .(2.15)$$

By substituting the technical relationships of production into the system of equations (2.6) to (2.10), the resulting system of equations becomes the basic system of linear equations describing the economy.

$$X_1 - a_{12}X_2 - a_{13}X_3 - a_{14}X_4 - a_{15}X_5 = y_1 \quad (2.16)$$

$$-a_{21}X_1 + X_2 - a_{23}X_3 - a_{24}X_4 - a_{25}X_5 = y_2 \quad (2.17)$$

$$-a_{31}X_1 - a_{32}X_2 + X_3 - a_{34}X_4 - a_{35}X_5 = y_3 \quad (2.18)$$

$$-a_{41}X_1 - a_{42}X_2 - a_{43}X_3 + X_4 - a_{45}X_5 = y_4 \quad (2.19)$$

$$-a_{51}X_1 - a_{52}X_2 - a_{53}X_3 - a_{54}X_4 + X_5 = y_5 \ . \quad (2.20)$$

This system of five linear equations and five unknowns can have a general solution if the matrix of the coefficients in the left-hand member is nonsingular (1, p. 55).

Hawkins and Simon (11, p. 247) have shown that the system of nonhomogeneous equations can have an economic meaning only if X_i's are all positive, and a necessary and sufficient condition for all X_i's to be positive is that all principal minors of the matrix A be positive.

The solution of the above linear system of equations, (2.16) to (2.20), gives a system of relationships expressing the respective net outputs as a function of the five parts of the final bill of goods. The coefficients of the solution express the relationships between the final bill of goods and net outputs. These coefficients will be referred to as the interdependence coefficients to distinguish them from the input coefficients. The algebraic solution of the basic system of equations (2.16) to (2.20) appears as follows:

$$A_{11}y_1 + A_{12}y_2 + A_{13}y_3 + A_{14}y_4 + A_{15}y_5 = X_1 \qquad (2.21)$$

$$A_{21}y_1 + A_{22}y_2 + A_{23}y_3 + A_{24}y_4 + A_{25}y_5 = X_2 \qquad (2.22)$$

$$A_{31}y_1 + A_{32}y_2 + A_{33}y_3 + A_{34}y_4 + A_{35}y_5 = X_3 \qquad (2.23)$$

$$A_{41}y_1 + A_{42}y_2 + A_{43}y_3 + A_{44}y_4 + A_{45}y_5 = X_4 \qquad (2.24)$$

$$A_{51}y_1 + A_{52}y_2 + A_{53}y_3 + A_{54}y_4 + A_{55}y_5 = X_5 . \qquad (2.25)$$

Each $A_{ij}y_j$ represents that part of the net output of the i-th sector which is due to the y_j units of commodity j entered in the final bill of goods. Thus $A_{12}y_2$ represents the part of the net output of the first sector which is due to the y_2 units of commodity 2 entered in the final bill of goods. In other words, it shows the amount of net output from the first sector which must flow to the second sector to establish the necessary net outputs from all sectors. $A_{22}y_2$ represents the direct contribution of the second sector to the final bill of goods.

The significance of each of the A_{ij}'s in the final solution is that they express the relationship between a change in the final bill of goods and the net output of the five sectors of the economy. If an increase in the portion of the final bill of goods supplied by the third sector is assumed, then A_{13}, A_{23}, A_{33}, A_{34} and A_{35} indicate by how much the net output of each sector must increase to supply the assumed final bill of goods.

The magnitude of the A_{ij}'s of the solution to the basic system of equations depends upon the magnitude of the technical coefficients of production. The matrix of technical coefficients of production constitutes the structural flow matrix.[1]

It characterizes the economy in that identical structural flow matrices represent identical economies if the aggregation is similar, and the sectors are identified with the same row vectors of the matrices.

$$A = \begin{vmatrix} 1 & -a_{12} & -a_{13} & -a_{14} & -a_{15} \\ -a_{21} & 1 & -a_{23} & -a_{24} & -a_{25} \\ -a_{31} & -a_{32} & 1 & -a_{34} & -a_{35} \\ -a_{41} & -a_{42} & -a_{43} & 1 & -a_{45} \\ -a_{51} & -a_{52} & -a_{53} & -a_{54} & 1 \end{vmatrix}$$

The inverse of the structural flow matrix appears as follows:

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[1]The element A^{ij} of the inverse matrix is obtained by dividing the cofactor A_{ji} of the structural flow matrix by the determinant $|A|$:

$$A^{ij} = \frac{A_{ji}}{|A|} .$$

$$A^{-1} = \begin{vmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} \end{vmatrix}$$

Alternative methods of solving for the inverse matrix are available to the investigator. The Doolittle Method (5) of inverting a matrix was used in this investigation.[1]

Employment required for any given output can also be determined if the labor input per unit of output is assumed to be constant. The following relationships between dollar value of labor used by each sector and the total net outputs are added to the system of equations (2.11) to (2.15).

$$x_{61} = a_{61}X_1;\; x_{62} = a_{62}X_2;\; x_{63} = a_{63}X_3;\; x_{65} = a_{65}X_5 \quad (2.26)$$

where x_{61}, x_{62}, x_{63}, x_{65} are the dollar value of labor consumed by the sectors 1 to 5, and a_{61}, a_{62}, a_{63}, a_{65} are the dollar value of labor per unit of net output.

To compute the dollar value of labor required in each sector for any given level of net output, the estimated net

[1]An excellent exposition of the Doolittle Method of inverting a matrix is given by Bancroft and Anderson (2, pp. 192-197).

output, X_j', is substituted into the appropriate formula in equation (2.26).[1] An estimate of the labor requirement for the first sector is given by the formula:

$$x_{61} = a_{61}X_1' \;. \tag{2.27}$$

The total employment of the economy is determined by summing the labor requirements of all sectors.

The dependence of the employment within a sector upon the final bill of goods can be shown by substituting the left-hand side of equation (2.21) for X_1' in equation (2.27).

$$x_{61} = a_{61}(A_{11}y_1' + A_{12}y_2' + A_{13}y_3' + A_{14}y_4' + A_{15}y_5') \tag{2.28}$$

or

$$\begin{aligned} x_{61} = {} & a_{61}A_{11}y_1' + a_{61}A_{12}y_2' + a_{61}A_{13}y_3' + a_{61}A_{14}y_4' \\ & + a_{61}A_{15}y_5' \end{aligned} \tag{2.29}$$

where y_j' denotes a new final bill of goods.

The analysis is limited to output, consumption, and employment with particular emphasis on the use of the methodology to analyze the relationship between two sectors of the agricultural economy and among sectors of the agricultural economy and other sectors. Leontief (17) has shown

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[1]X_j' refers to the net output of the j-th sector at a different point in time.

that input-output schemes may be applied to analyses of investment, wages, prices, profits, and other macroeconomic problems.

III. THE ECONOMIC MODEL

A detailed study of the United States economy would involve an extremely large number of variables and would consist of such a large number of equations that it would be impracticable to solve the system. Since it is impossible to deal with all the variables of a complex economy such as found in the United States, it is necessary to aggregate the economy into something more managable.[1] The economy is thus conceived as consisting of n defined sectors, each producing a similar but not necessarily homogeneous product (23, p. 132).

In any particular input-output study, several factors may limit the number of sectors included in the model. The amount of resources which the investigator has at his disposal for collecting data and performing the calculations to solve the system may limit the number of sectors. Computational equipment may not be available to handle large matrices resulting from the larger models. The data may not be

[1]Mathematical computations used in input-output analysis cause a multiplication of the errors which becomes more serious as the order of the matrix increases.

available and reliable estimates may be impossible.

The aggregation of the economy, however necessary, adds a difficult problem to the input-output technique because the same data aggregated to a model of n_1 sectors and alternately to a model of n_2 sectors may not give the same results when examining the interdependence of the sectors. The question as to which is the most reliable answer on which to make the inference concerning the interdependence becomes a problem of multiple choice. Leontief suggests a controlled experiment or a direct observation of the economy to test the validity of the results (22, p. 207). The problem of aggregation is worthy of much further investigation in determining appropriate models for input-output analyses.[1] Over-simplification may be an abuse of the methodology, but the smaller model should serve a useful purpose in illustrating the use of the input-output technique in agricultural production research.

This study was designed specifically for determining the usefulness of the Leontief input-output analysis in studying aggregate components of the agricultural economy.

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[1]Tintner (28) discusses the use of the method of principal components as a tool for dealing with the aggregation problem. He shows how factor analysis is applicable to "replacing several variables with a few principal components" such as is done in the derivation of index numbers and the "transition from microeconomic to macroeconomic analysis". (28, pp. 102-114).

The data are totals of the annual flows of net output among five sectors of the economy. In the open model, labor service is treated as a primary factor of production and is used to examine the change in employment associated with assumed levels of output and consumption. The inputs to households (consumption) become the final bill of goods and are variables determined outside the system.

A. The Equations of the System

Two equations have been used to describe agriculture. The first equation in the system pertains to what has been termed primary agricultural production.[1] Primary agricultural production includes crop production and all other production where the products are harvested directly from the culture of plant life. The second equation in the system pertains to what has been called secondary agricultural production. This includes all agricultural production

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[1]Heady (12, p. 743) in discussing resource productivity of primary production points out that agriculture is not strictly an "extractive" industry but also processes primary materials. Livestock production thus falls into the classification of secondary industries. The terms primary crop production and secondary livestock production are frequently found in the literature.

resulting from the processing of crops through livestock, and storage activities conducted on farms. Storage of grain on farms and in bins owned by the Commodity Credit Corporation was included in the second equation. Grain storage on farms and in government owned bins was treated as secondary production because it was assumed that stored grain had reached its end in the crop production process, but both sources of grain storage are available to farmers for later production of livestock or may be held purely for speculative purposes.

The third equation describes the production of all industries and services. The aggregation within the industry and services sector becomes more serious than that used in the Leontief study; however, a lower degree of aggregation is used in the agricultural sectors. This may be an important limitation of the model since the industrial sector contributes the greatest portion to the final bill of goods and contributes most to employment in the United States economy. Government enterprises, such as public service, etc., are included in this equation. It may be wise to include more industrial equations and an unallocated sector in future studies to reduce the influence of the industrial aggregation. Again the multiple choice problem must be dealt with here in determining which model gives the most reliable result. So far, no empirical methods are available, and it thus falls to the experience and judgment of the investigator as well as available funds to determine the choice of the economic model.

The fourth equation treats foreign trade as a sector using inputs and producing outputs. Exports are the inputs or flows from other sectors to the foreign trade sector, and imports are the outputs of the sector.

A problem arises in input-output models as to how to allocate imports among the other sectors of the economy. Two alternative methods are available: 1. Imports can be allocated to the sectors which produce similar products. For example, imports of agricultural products would be allocated to the agricultural sector and added to the net output of the sector. 2. Imports can be allocated directly to the sectors which use them.

Leontief (22, p. 164) used the first method. He argues that the technical structures of the sectors of the economy are determined by ratios of inputs to outputs regardless of the origin of the inputs. This method does not separate domestic production from foreign production when the results of the input-output analysis are used to guide policies affecting domestic production.

The second method was used in this study. Imports are allocated directly to the sectors which use them. Cotton produced by domestic agriculture is included in the input coefficient of primary agricultural products to industry. Imported cotton is included in the input coefficient of foreign trade to industry. This method of allocating imports does separate domestic production from foreign production; therefore, the results of input-output analysis may be more useful in determining policies affecting domestic production.

Exports are not included in the fourth equation but appear as components in other equations. In determining what commodities flow from other sectors to the foreign trade sector, a decision must be made regarding the flow of export commodities. That is, do agricultural exports flow directly from the agricultural sector to foreign trade, or do they first pass through wholesale firms? In reality, all products involved in foreign trade do pass through wholesale firms. Agricultural exports could be considered inputs to industry and services sector; however, then the input-output analysis would not show any relationship between agriculture and foreign trade. Since this is an important consideration in policy making, the quantity of net output of a sector flowing to the foreign trade sector was considered to be the physical net output which ultimately reaches the export market without any appreciable change in form. Services performed by firms and households (labor) in carrying out the activities of foreign trade were included in the industry and services sector.

To complete the input-output system, government activities are described by the fifth equation. Government is treated similarly to any other sector of the economy. Output of government services is measured by government receipts, and inputs to government are measured by government expenditures, both exclusive of receipts and payments of government enterprises.

The sixth sector of the economy constitutes the final demand. Consumption by households has been included in final demand; however, other goods and services such as those which are used for new plants and equipment (net investment) could be included. The assumed static economy in the model for this study allocates the physical output used for capital to the individual sector rather than to final demand. The final demand for goods of consumption are the variables determined outside the system and are the constants of the first five equations, thus no equation appears in the system for the output of households. Dollar value of labor services consumed by each sector could be treated as the output of households, but in the open system labor services are used only for determining the technical input coefficients for labor. Profits and interest payments, which constitute payments to households for services other than labor, have been omitted from the data.

B. Description of the Variables

The variables of the five equations which correspond to equations (2.1) to (2.5) and labor services which correspond to equation (2.26) included in the economic model are as follows:

	x_{12}	x_{13}	x_{14}	x_{15}	y_1	X_1	(3.1)
x_{21}		x_{23}		x_{25}	y_2	X_2	(3.2)
x_{31}	x_{32}		x_{34}	x_{35}	y_3	X_3	(3.3)
		x_{43}		x_{45}	y_4	X_4	(3.4)
x_{51}	x_{52}	x_{53}	x_{54}		y_5	X_5	(3.5)
x_{61}	x_{62}	x_{63}		x_{65}			(3.6)

Where x_{ij} is not included in the equation, it is assumed that no products of the i-th sector are used in the production process of the j-th sector.

Thus:

$$x_{ij} = 0 \ .$$

Data representing the activity of the economy during a single year have been aggregated into the variables included in system of equations. A general description of the variables is given here in order to summarize the economic model without becoming involved in a detailed description of the sources of data for estimating the variables and of the procedures followed in collecting the data. More detail as to sources and estimating procedures is given in the appendix.

1. Primary agricultural production

The variables of primary agricultural production are x_{12}, x_{13}, x_{14}, x_{15}, y_1, X_1 where

x_{12} is the value of all feed fed to livestock including farm grown grains, forage, hay, pasture, and the net increase in stocks of grain stored on farms and in bins owned by Commodity Credit Corporation.

x_{13} is the value of all crops sold to industry and the value of forest products.

x_{14} is the value of all exported crops not processed other than preparing them for shipment.

x_{15} is the value of crops purchased by government, generally indicated as government procurement, conservation payments to farmers, payments for naval stores, and payments under the Sugar Act.

y_1 is the value of crops consumed by farm households and crops exchanged for consumption goods.

X_1 is the summation of all the other variables in the equation.

2. <u>Secondary agricultural production</u>

The variables of secondary agricultural production are

x_{21}, x_{23}, x_{24}, y_2, X_2

where

x_{21} is the value of manure produced by livestock.

x_{23} is the value of all livestock sold to industry for slaughter.

x_{24} is the value of livestock exported and all raw livestock products exported but not processed except for preparing them for shipment.

y_2 is the value of livestock and livestock products consumed on farms and livestock products exchanged directly for consumption goods.

X_2 is the summation of all other variables in the equation.

3. Industry and services

The variables of industry and services are x_{31}, x_{32}, x_{34}, x_{35}, y_3, X_3
where

x_{31} is the value of fertilizer, repairs, fuel and oil, machinery and equipment, and seeds purchased by farmers.

x_{32} is the value of commercial feeds fed to livestock, value of new construction of buildings and fences, electric power, and veterinary services and supplies.

x_{34} is the total value of all exports of goods produced by industry.

x_{35} is the value of government purchases of goods and services from business less purchases of agricultural products through government procurement and purchases by government enterprises.

y_3 is the value of all personal expenditures for durable and non-durable goods and services except those which are accounted for in other variables of the final bill of goods.

X_3 is the summation of all other variables in the equation.

4. Foreign trade

The variables of foreign trade are x_{43}, x_{45}, y_4, X_4 where

x_{43} is the value of all imports consumed by industry.

x_{45} is the value of government purchases from abroad.

y_4 is the value of imports of crude food-stuffs, manufactured food-stuffs, and finished manufactures purchased by households.

X_4 is the value of all general imports.

5. Government

The variables of government are x_{51}, x_{52}, x_{53}, x_{54}, y_5, X_5
where

x_{51} is all farm real estate taxes paid to government, and a proportionate share of farm personal property taxes paid to government. Personal property taxes on machinery and equipment was assumed to be a measure of the flow from government to primary agricultural production.

x_{52} is a proportionate share of farm personal property taxes paid to government. Personal property taxes on livestock and inventories of feed was assumed to be a measure of the flow from government to secondary agricultural production.

x_{53} is total government receipts from corporate taxes, indirect business taxes, and property taxes chargeable to business.

x_{54} is the value of government sales abroad and government revenues from customs.

y_5 is all personal taxes and all non-tax revenues not chargeable to business and the total contribution of employees to social insurance.

X_5 is the total of government receipts less contributions of employers to social insurance. The contributions of employers to social insurance were assumed to be a payment to individuals for labor services and, therefore, were added to salaries and wages.

6. Household (labor)

Although no equation is used to describe the total output of labor services, coefficients of labor input are obtained from the value of labor services consumed by each of the foregoing five sectors. The variables of labor services are x_{61}, x_{62}, x_{63}, x_{65}. These correspond to the variables in equation (2.26).

x_{61} is the value of all labor used in the production of crops plus a proportionate share of the value of labor used for farm maintenance plus wages, salaries, and supplements of forestry workers.

x_{62} is the value of all labor used in the production of livestock plus a proportionate share of the value of labor used for farm maintenance.

x_{63} is the total wages, salaries, and supplements of industry and services plus wages, salaries and supplements of government enterprises.

x_{65} is the wages and salaries plus supplements to wages and salaries of government employees.

IV. THE EMPIRICAL SOLUTION

A. Input-output Flow and the Coefficients of Production

Presently available data[1] for aggregate estimates of inputs and outputs of the five sectors of the United States economy were collected and formulated into input-output flow tables given in Tables 1, 2, and 3 for 1949, 1939, and 1929, respectively.[2] Each row in the flow table shows the distribution of the net output of a sector among all other sectors of the economy, including the household sector of the economy. Each column in the flow table shows the inputs to a sector from other sectors of the economy, including the input of labor services from households. For example, in Table 1, 697,444 thousands of dollars' worth of the net output of secondary agricultural production was used by primary agricultural production to produce a net output

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[1]See Appendix, p. 111, for sources of the data used to estimate the variables of the system.

[2]The accuracy of the data in Tables 1, 2, and 3 is not to be over-estimated by the reader. National income statistics reported by the Department of Commerce are given in millions. Calculations were made on the basis of the figures given in the tables; however, it has been pointed out, that the additional digits do not improve the accuracy of the results.

Table 1. Distribution of Inputs and Outputs of the United States Economy, 1949 (thousand dollars)

Sectors producing the net output	Sectors consuming the net output						
	Primary agricultural production	Secondary agricultural production	Industry and services	Foreign trade (exports)	Government	Household consumption	Net output
Primary agricultural production		11574758	7297831	1794681	526818	601157	21795245
Secondary agricultural production	697444		15304052	39719		1720648	17761863
Industry and services	7330000	2578944		10586650	19987635	175682341	216165570
Foreign trade (imports)			4635683		4146000	1986659	10768342
Government	767657	55552	31628244	693000		21565000	54709453
Households labor	7759240	4980320	117309000		20424000		x x x
Total outlays	17009586	22304464	176174810	13114050	45084453	201555805	x x x

Table 2. Distribution of Inputs and Outputs of the United States Economy, 1939 (thousands of dollars)

Sectors producing the net output	Sectors consuming the net output						
	Primary agricultural production	Secondary agricultural production	Industry and services	Foreign trade (exports)	Government	Household consumption	Net output
Primary agricultural production		3882033	1752704	433314	763000	448764	7279815
Secondary agricultural production	373338		4464344	6694		760813	5605189
Industry and services	1865000	910252		2678335	5860000	65537367	76850954
Foreign trade (imports)			1522465		69000	684633	2276098
Government	425744	25770	9242486	604000		3036000	13334000
Households labor	2542061	1590646	39122000		7629000		x x x
Total outlays	5375146	7609499	127078045	3722343	14321000	70467577	x x x

Table 3. Distribution of Inputs and Outputs of the United States Economy, 1929 (thousands of dollars)

Sectors producing the net output	Sectors consuming the net output: Primary agricultural production	Secondary agricultural production	Industry and services	Foreign trade (exports)	Government	Household consumption	Net output
Primary agricultural production		6496585	2776482	1266276		800458	11339801
Secondary agricultural production	440740		5332003	16754		903542	6693039
Industry and services	2368000	1200382		3841053	3899000	75747870	87056305
Foreign trade (imports)			2982231		103000	1314130	4399361
Government	593156	47660	8713184	377000		2785000	12516000
Households labor	4121133	2294022	44851000		4356000		x x x
Total outlays	7766574	12086592	73935595	5501083	8358000	81551000	x x x

of 21,795,245 thousands of dollars during the year 1949.

The part of total output used by the sector which produced it is not included in the input-output flow table. The diagonal elements in the flow table are eliminated because the mathematical model provides an analysis of only the flow of resources among the sectors. The input-output flow tables were constructed from physical quantities of goods and services flowing among the sectors and the final bill of goods (households). All physical quantities are expressed in terms of dollar value and may be thought of as the units of physical quantities purchasable with 1000 dollars. Deriving the net output by addition of the dollar value of output consumed by each sector necessitates converting the physical units to a common unit of measurement. A stationary economy was assumed to eliminate the problem of capital formation. Investments to replace or maintain plants and equipment are included in the flows from one sector to another. Interest payments and profits could be included in the household row of the input-output flow table; however, these items do not come into the system of equations, therefore only labor services are of concern in the household row in constructing the table.

A major portion of the resources devoted to input-output analysis is represented in the construction of the input-output flow table. The details of constructing the tables from available sources are given in the appendix. These tables furnish the bases for deriving the constant input coefficients of production needed for setting up the basic system of equations for a Leontief input-output model of the economy.

The technical coefficients of production were derived by dividing each element of the columns in Tables 1, 2, and 3 by corresponding net output of the sector given in the last column of the table. Thus each element in the first column was divided by the first element in the last column to obtain the dollar's worth of input per dollar's worth of net output of the first sector (primary agricultural production). For example, the 1949 input coefficients for primary agricultural production are .03200 for secondary agricultural products, .33631 for industry and services products, and .03522 for government services. These were derived by dividing 697,444 thousands of dollars' worth of secondary agricultural products, 7,330,000 thousands of dollars' worth of industry and services products, and 767,657 thousands of dollars' worth of government services by the total net output of primary agricultural production, 21,795,245 thousands of dollars.

This procedure was followed to compute all the technical production coefficients for all sectors for 1949, 1939, and 1929. The results are given in Tables 4, 5, and 6.

Klein (19) questions the interpretation of the input coefficients. He raises the question as to whether the Leontief input-output coefficients are purely technical parameters or simply ratios of two economic variables. He concludes that if the model assumes joint production and market competition then:

> In general the elements of Leontief's input-output table may be interpreted as parameters of a class of production functions, all of which permit substitution among factors of production and types of output. Moreover, the a_{ik} (production coefficient) can be interpreted as technological parameters. (19, p. 134)

He further states that in the imperfect market the a_{ik} is dependent upon production, demand, and supply functions which would cause the a_{ik} to change and therefore cannot be interpreted as technological relations. This he concludes is the more realistic interpretation of the Leontief system.

Samuelson (24), Koopmans (20), and Arrow (3) have shown that Leontief's theory is compatible with the more general case of substitutability.

Table 4. Technical Input Coefficients and Consumption of the United States Economy, 1949

Sectors producing the net output	Sectors consuming the net output					
	Primary agricultural production	Secondary agricultural production	Industry and services	Foreign trade	Govern-ment	Household (consumption)
Primary agricul-tural production		.65166	.03376	.16666	.00963	601,157
Secondary agricul-tural production	.03200		.07080	.00369		1,720,648
Industry and services	.33631	.14520		.98313	.36534	175,682,341
Foreign trade			.02145		.07578	1,986,659
Government	.03522	.00313	.14631	.06436		21,565,000
Households (labor)	.35601	.28039	.54268		.37332	

Table 5. Technical Input Coefficients and Consumption of the United States Economy, 1939

Sectors producing the net output	Sectors consuming the net output					
	Primary agricultural production	Secondary agricultural production	Industry and services	Foreign trade	Govern-ment	Household (consumption)
Primary agricul-tural production		.69258	.02281	.19038	.05722	448,764
Secondary agricul-tural production	.05128		.05809	.00294		760,813
Industry and services	.25619	.16239		1.17672	.43948	65,537,367
Foreign trade			.01981		.00517	684,633
Government	.05848	.00460	.12027	.26537		3,036,000
Households (labor)	.34919	.28378	.50906		.57215	

Table 6. Technical Input Coefficients and Consumption of the United States Economy, 1929

Sectors producing the net output	Sectors consuming the net output					
	Primary agricultural production	Secondary agricultural production	Industry and services	Foreign trade	Govern- ment	Household (consumption)
Primary agricul- tural production		.97065	.03189	.28783		800,458
Secondary agricul- tural production	.03887		.06125	.00381		903,542
Industry and services	.20882	.17935		.87309	.31152	75,747,870
Foreign trade			.03426		.00823	1,314,130
Government	.05231	.00712	.10009	.08569		2,785,000
Households (labor)	.36342	.34275	.51520		.34804	

The proof of the Samuelson's theorem (24, p. 143) is beyond the scope of this exposition, but it is important to recognize that the Leontief model "has a greater generality than a literal reading of its assumptions suggests," (20, p. 147).

The last columns in Tables 4, 5, and 6 are the direct contribution of each sector to the final bill of goods. The model used in this study included only household consumption of goods and services in the final bill of goods. The final bill of goods measures the dollars' worth of goods and services contributed toward final consumption by each sector from the net output of the sector.

B. The Basic System of Equations

From the information in Tables 4, 5, and 6 the basic system of equations of the Leontief technique can be constructed. The first five columns in each table together with the diagonal elements represents the structural flow matrix of the economy. The elements of the household column are the variables determined outside the system of equations.

The three basic systems of equations for 1949, 1939, and 1929 are as follows:

$$X_1 - .65166\,X_2 - .03376\,X_3 - .16666\,X_4 - .00963\,X_5 = 601{,}757 \tag{4.1}$$

$$-.03200\,X_1 + X_2 - .07080\,X_3 - .00369\,X_4 = 1{,}720{,}648 \tag{4.2}$$

$$-.33631\,X_1 - .14520\,X_2 + X_3 - .98313\,X_4 - .36534\,X_5 = 175{,}682{,}341 \tag{4.3}$$

$$-.02145\,X_3 + X_4 - .07578\,X_5 = 1{,}986{,}659 \tag{4.4}$$

$$-.03522\,X_1 - .00313\,X_2 - .14631\,X_3 - .06436\,X_4 + X_5 = 21{,}565{,}000 \tag{4.5}$$

$$X_1 - .69258\,X_2 - .02281\,X_3 - .19038\,X_4 - .05722\,X_5 = 448{,}764 \tag{4.6}$$

$$-.05128\,X_1 + X_2 - .05809\,X_3 - .00294\,X_4 = 760{,}813 \tag{4.7}$$

$$-.25619\,X_1 - .16239\,X_2 + X_3 - 1.17672\,X_4 - .43948\,X_5 = 65{,}537{,}367 \tag{4.8}$$

$$- .01981\ X_3 + X_4 - .00517\ X_5 = 684{,}633 \qquad (4.9)$$

$$- .05848\ X_1 - .00460\ X_2 - .12027\ X_3 - .26537\ X_4 + X_5 = 3{,}036{,}000 \qquad (4.10)$$

$$X_1 - .97065\ X_2 - .03189\ X_3 - .28783\ X_4 = 800{,}458 \qquad (4.11)$$

$$- .03887\ X_1 + X_2 - .06125\ X_3 - .00381\ X_4 = 903{,}542 \qquad (4.12)$$

$$- .20882\ X_1 - .17935\ X_2 + X_3 - .87309\ X_4 - .31152\ X_5 = 75{,}747{,}870 \qquad (4.13)$$

$$- .03426\ X_3 + X_4 - .00823\ X_5 = 1{,}314{,}130 \qquad (4.14)$$

$$- .05231\ X_1 - .00712\ X_2 - .10009\ X_3 - .08569\ X_4 + X_5 = 2{,}785{,}000 \qquad (4.15)$$

C. The Solution

The inverse of the structural flow matrix of each basic system of equations is given in Tables 7, 8, and 9. The coefficients of the inverse matrix furnish the

Table 7. Interdependence Coefficients Between the Final Bill of Goods and Net Outputs for 1949

Sectors producing the net output	Sectors consuming the net output				
	Primary agricultural production	Secondary agricultural production	Industry and services	Foreign trade	Government
Primary agricultural production	1.05925	.70528	.10186	.28372	.06891
Secondary agricultural production	.06356	1.05383	.08463	.10020	.03912
Industry and services	.41816	.44070	1.14734	1.23254	.51660
Foreign trade	.01653	.01655	.03780	1.04598	.09324
Government	.09975	.09368	.17416	.25796	1.08414

Table 8. Interdependence Coefficients Between the Final Bill of Goods and Net Outputs for 1939

Sectors producing the net output	Sectors consuming the net output				
	Primary agricultural production	Secondary agricultural production	Industry and services	Foreign trade	Government
Primary agricultural production	1.06679	.75341	.08686	.33430	.10095
Secondary agricultural production	.07450	1.06311	.06989	.10898	.03554
Industry and services	.34039	.42088	1.12526	1.52864	.52192
Foreign trade	.00729	.00887	.02305	1.03276	.01589
Government	.10560	.10192	.14685	.47796	1.07305

Table 9. Interdependence Coefficients Between the Final Bill of Goods and Net Outputs for 1929

Sectors producing the net output	Sectors consuming the net output				
	Primary agricultural production	Secondary agricultural production	Industry and services	Foreign trade	Government
Primary agricultural production	1.06765	1.05760	.11711	.41699	.03991
Secondary agricultural production	.05795	1.06945	.07264	.08617	.02334
Industry and services	.26811	.46170	1.10922	1.07776	.35442
Foreign trade	.00988	.01673	.03899	1.03872	.02070
Government	.08394	.11058	.12100	.21931	1.03950

information for observing the interdependence of changes in the final bill of goods and the net outputs of all sectors.

D. Interpretation of the Results

One purpose of this study is to examine the possibility of adapting the Leontief input-output analysis as a technique for analyzing economic problems in agriculture. To determine whether input-output analysis provides any information to the economist, the results will be examined for evidences of previously established phenomena and relationships in agricultural production. Other economic interpretations will be directed to the economic meaning of the interdependent relationships revealed by the input-output analysis. The analysis and implications will be divided into two parts: (1) a general consideration of the interdependence coefficients and (2) specific considerations of the input-output analysis applied to problems in agriculture.

E. A General Consideration

The interdependence coefficients in Tables 7, 8, and 9 might first be examined in general to determine the extent of interdependence among the sectors of the economy.

The coefficients in the primary agricultural column of Table 7 indicate a considerable degree of interdependence between primary agricultural production and the industrial sector. This is what one would suspect, since agricultural crop production is highly dependent upon fertilizer, machinery, fuel and other sizable inputs from industrial sources. A marked relationship between secondary agricultural production and primary agricultural production is also evident in Table 7. This is logical since feeds are the most important input to livestock production. Industrial interdependence with secondary agricultural production is also of importance because commercial feeds make up an important part of the total feed consumed by livestock. All other interdependence coefficients are much less important in the agricultural economy.

The large aggregation in the industry and services sector may account for the small coefficients of the industry and services column in Table 7. It would appear that industry is more intradependent with industries within the industrial sector than it is interdependent with the sectors included in this study. Evidence to test this hypothesis is not available from this study since only one equation was used for all industries and services. Government is the most interdependent with industry and service of those sectors included in the model. From Table 1, it can be noted that the payment to government from industry and services exceeds the combined flow of primary and secondary agricultural commodities to industry.

The foreign trade column of Table 7 is somewhat more difficult to interpret. The treatment of foreign trade as a sector producing an output of imports and using exports as inputs is not very realistic. The input-output Tables 1, 2, and 3 indicate that the United States had a credit import balance of physical commodities in all three years. The dollar value of inputs to foreign trade treated as an industry exceeded the total net output. This could also occur in any other sector in a single year if new capital equipment was being added or inventories were increasing but would not continue for a long period of time such as might occur in the foreign trade sector. The effect of a credit import balance is illustrated in the input-output table for 1939 (Table 2). The total exports from the industry and services sector was greater than the total value of imports, thus giving an input coefficient greater than 1 (Table 5) in the foreign trade sector. The interdependence coefficients of foreign trade (Table 7) would indicate that an increase by one dollar's worth in the direct demand for imports would result in an increase of the net outputs by .283 dollar's worth of primary agricultural products, by .100 dollar's worth of secondary agricultural products, by 1.232 dollar's worth of industrial products and services, and by .257 dollar's worth of government receipts. This implicitly assumes that the same balance of trade would be maintained, otherwise the input coefficients of foreign trade could not be assumed constant. The interdependence coefficients of foreign trade in the results of this study are not appropriate for determining the effects of most international trade policies. The input coefficients maintain a status quo in the balance of trade, whereas policy is more likely to be directed at changing the balance of trade through tariff regulation and trade agreements.

It is interesting to note that the interdependence coefficients of foreign trade and government seem to reflect the increasing importance of government sales abroad. Comparing the coefficients within each year, the increase in the volume of government sales abroad during the prewar period, 1939, is evident in the interdependence coefficients. The interdependence coefficient between foreign trade and industry and services may seem rather unrealistic since it indicates that a one dollar's worth of increase in the direct demand for imports would necessitate a 1.232 dollar's worth of net output from industry and services. At first it may appear as though this increase is necessary because imports are processed by industry before reaching the consumer. This is not the case. When imports are regarded as an output of foreign trade, it means that the industrial sector is an important supplier of export commodities since in order to bring forth the additional import to supply the increased demand, a sizable increase in export goods must be forthcoming from the industrial sector.

A much more realistic analysis of the effects of changes in foreign trade could be obtained from an input-output analysis in which the foreign trade equation is removed from the system and exports are included in the final bill of goods. This procedure was followed by Leontief in a study of foreign trade, domestic output and employment (22, p. 163).

The coefficients in the government column of Table 7 indicate the greatest interdependence occurring between government and industry and services. This is to be expected since purchases by government from industry and services is the largest flow of payments from government except payments to households for which there is no interdependence coefficient in the present model.

F. Changes in Consumption, Output, and Employment

The previous section was concerned with a general inspection of the interdependence coefficients derived by a solution of a system of equations describing the production of related sectors of the economy. The inferences were based on a direct comparison of the magnitude of the coefficients within a single sector. Question now arises as to what meaning the coefficients have and why the magnitude is an indication of the interdependence. These questions can best be answered by assuming changes in consumption and observing the effects on net outputs and employment of all sectors; by assuming changes in output and observing the change in resource requirements (inputs); and by assuming changes in outputs and consumption and observing the effects on employment.

1. Changes in consumption

Several aspects of changes in consumption are of interest in agricultural production economics: (1) What change in net outputs of the agricultural sectors are necessary with changes in the final bill of goods? (2) What flows of resources from other sectors are necessary for agriculture to produce these net outputs? (3) What flows of resources among sectors of agriculture are necessary to meet consumption requirements?

The dollar's worth of change in the net outputs per dollar's worth of change in the final bill of goods can be examined by using the solution of the basic system of equations describing the economy. The interdependence coefficients between net outputs and the final bill of goods are given in Tables 7, 8, and 9. In 1949 a one dollar's worth of increase in the direct demand for secondary agricultural products would necessitate a .705 dollar's worth of increase in the output of primary agricultural production, a .440 dollar's worth of increase in the output of industry and services, a .016 dollar's worth of increase in imports, and a .093 dollar's worth of increase in government receipts.

This prediction necessarily assumes that the basic relations among the sectors of the economy existing in 1949 continue unchanged. This, however, may not be the case and the inference would be subject to error dependent upon changing technology and the validity of linear production relationships as well as errors of observation. The inferences can be extended to the other sectors of the economy by observing the coefficients in the respective columns of Table 7. Since this study is primarily concerned with interpretations of the agricultural effects, the discussion will be limited to interpretation of interdependent relationships involving agriculture.

The effects of changes in direct demand for agricultural products in the Leontief system are not of much significance. Consumption of farm products in farm households are only a small part of the agricultural products consumed by all households. A declining farm population has reduced the quantities of farm products consumed in farm households. An increasing level of prosperity such as occurred after 1939 would diminish the tendency of farm people to depend on home gardens and farm slaughter of livestock for an important part of their diet thus further reducing direct demand. More important to agricultural policy are the effects of derived demand for agricultural products. These effects are observed by examining the first two rows of Table 7. If an increase in the direct demand by one dollar's worth of industrial and services' products is assumed to come about by increased population, the flow of primary agricultural products to industry and services must increase .101 dollar's worth per one dollar of increase in demand, and the flow of secondary agricultural products to industry and services must increase .084 dollar's worth per dollar of increase in demand.

An important area of analysis in production economics is the problem of increasing production to maintain the standard of living for an increasing population or a progressive society. Particular concern is often expressed over the possibility of food production falling short of that necessary to provide a continued or increased standard of living. Resource requirements of sectors producing additional food supplies as well as other consumption goods for an increasing population can be analyzed with the aid of input-output studies. Table 10 shows the per cent change in net output of each sector associated with a 10 per cent change in each part of the final bill of goods. In 1949, a 10 per cent increase in the direct demand for primary agricultural products would necessitate a .292 per cent increase in total net output of primary agricultural production. To produce this increase in primary agricultural production, additional resources must flow from other sectors supplying inputs to primary agriculture. Net output of secondary agricultural production would need to increase by .022 per cent; net output of industry and services would need to increase by .012 per cent; imports would need to increase by .0C9 per cent; and government services would need to increase by .011 per cent.

Proportionate JAS

Table 10. ~~Per Cent~~ of Change in Net Output of Sectors Associated with a 10 Per Cent Change in Each Portion of the Final Bill of Goods[1]

Sectors producing the net output	Year	Sectors supplying the final bill of goods: Primary agricultural production	Secondary agricultural production	Industry and services	Foreign trade	Government
Primary agricultural production	1949	.00292	.00557	.08211	.00259	.00682
	1939	.00658	.00787	.07820	.00314	.00421
	1929	.00754	.00843	.07823	.00483	.00098
Secondary agricultural production	1949	.00022	.01021	.08371	.00112	.00475
	1939	.00060	.01443	.08172	.00133	.00192
	1929	.00069	.01444	.08221	.00169	.00097
Industry and services	1949	.00012	.00035	.09324	.00113	.00515
	1939	.00020	.00042	.09596	.00136	.00206
	1929	.00025	.00048	.09651	.00163	.00113
Foreign trade	1949	.00009	.00026	.06167	.01930	.01867
	1939	.00014	.00030	.06637	.03106	.00212
	1929	.00029	.00034	.06713	.03102	.00131
Government	1949	.00011	.00029	.05593	.00094	.04273
	1939	.00035	.00058	.07217	.00245	.02443
	1929	.00054	.00079	.07323	.00230	.02313

[1]Per cent calculated by dividing data in Table 11 by the corresponding net outputs (Tables 1, 2, and 3).

Primary and secondary agriculture do not make appreciable direct contributions to the final bill of goods, thus effects of increased consumption of goods and services resulting from increasing population can be observed by assuming a 10 per cent increase in the portion of the final bill of goods contributed by industry and services and then observing the required resource flows from agriculture. In Table 10, a 10 per cent increase in demand for industry and services products, which includes processed food products, would necessitate an 8.21 per cent increase in primary agricultural net output and an 8.37 per cent increase in secondary agricultural net output.

The absolute change in dollar value associated with a 10 per cent change in the final bill of goods is given in Table 11. In 1949, a 10 per cent increase in the portion of the final bill of goods produced by industry and services would require approximately 1.8 billion dollars' worth of additional primary agricultural production and approximately 1.5 billion dollars' worth of additional secondary agricultural production. This would be comparable to one-half the corn crop produced in 1949 and one-third the value of all cattle sold for slaughter in 1949.

One problem confronting agricultural policy makers is how these increases in output can be brought about in order

Table 11. Absolute Change in Net Output of All Sectors Associated with a 10 Per Cent Change in Each Portion of the Final Bill of Goods (thousands of dollars)[1]

Sectors supplying inputs	Year	Sectors consuming inputs in producing net output: Primary agricultural production	Secondary agricultural production	Industry and services	Foreign trade	Government
Primary agricultural production	1949	63,677	121,353	1,789,500	56,365	148,604
	1939	47,873	57,320	569,257	22,887	30,648
	1929	85,460	95,558	887,083	54,797	11,114
Secondary agricultural production	1949	3,820	181,327	1,486,799	19,908	84,362
	1939	3,343	80,882	458,040	7,461	10,789
	1929	4,638	96,629	550,232	11,323	6,500
Industry and services	1949	25,137	75,828	20,156,737	244,863	1,114,047
	1939	14,275	32,021	7,374,657	104,655	158,454
	1929	21,461	41,716	8,402,105	141,631	98,705
Foreign trade	1949	993	2,847	664,079	207,800	201,072
	1939	327	674	151,063	70,706	4,824
	1929	790	1,511	295,340	136,502	5,764
Government	1949	5,996	16,119	3,059,683	51,247	2,337,947
	1939	4,738	7,754	962,350	32,722	325,777
	1929	6,719	9,991	916,549	28,820	289,500

[1]Computed by multiplying 10 per cent of the final bill of goods (Tables 4, 5, and 6) by the interdependence coefficients (Tables 7, 8, and 9).

to meet the demands of an increasing population or a progressive society. Input-output analysis assumes that the resources needed for increasing net output are available to the sectors of the economy. This would not be the case for all resources, especially land and capital. Heady (12, p. 798) states that agricultural output could always be increased by drawing more resources into agriculture; however, the problem of food production is not serious in the United States economy, since agricultural output could be increased considerably by adapting techniques already known. The latter method of increasing agricultural production would result in changes in the production coefficients which are assumed constant in the input-output technique used in this study.

If the increases in net outputs of the agricultural sectors are accomplished by technical change, the changes in input coefficients for primary and secondary agricultural production expressed in 1939 dollars might be extrapolated into the future, and the required resource flows estimated for total net output of the sector, including any expected increase in net output.

The coefficient for the input of secondary agricultural products to primary agricultural production (Table 16) had an average annual rate of change from 1939 to 1949 of 2.8

per cent in terms of 1939 dollars. This rate of change extended to 1975 would result in an input coefficient of .00913. Estimates of population increases and disposable income increases have been made by the United States President's Materials Policy Commission (41, p. 64) which indicate that a 30.3 per cent increase in primary agricultural production would be necessary to supply the demand for food products in 1975. Using this percentage increase in the net output and using the extrapolated input coefficient for primary agricultural production, net output of secondary production would need to increase by .29 per cent to supply the additional needs of the primary agricultural sector to produce the 30.3 per cent increase in net output. If the 1949 coefficient was assumed to hold for 1975, the increase in secondary production would be 1.18 per cent.

The President's Policy Commission's estimates also indicate a need for a 37.9 per cent increase in secondary agricultural production to supply the 1975 demand for food products. The coefficient for the input of primary agricultural products to secondary agricultural production (Table 16) had an average annual rate of change from 1939 to 1949 of 1.9 per cent in terms of 1939 dollars. The extrapolated coefficient for 1975 would be .28435. A37.9 per cent increase in secondary agricultural net output would necessitate an increase in primary agricultural net output of 10.2 per cent. If the 1949 coefficient was assumed to hold for 1975, the increase in primary agricultural production would be 20.1 per cent.

Extrapolation of the coefficients of inputs to agriculture from the industry and services sector becomes somewhat meaningless since the average annual rate of change is high, and both coefficients for agriculture are increasing. If the coefficients increased at the same rate, the increase in the magnitude of the flow of resources from industry to agriculture would not be offset by any technical change as is the case where the coefficient is decreasing. An increase in agricultural net output would always necessitate a greater total flow of resources from industry to agriculture than was observed by the 1949 input-output analysis. For this reason, a discussion of resource flow from industry to agriculture will be left to the discussion of changes in output.

The United States President's Materials Policy Commission (41) estimated that gross product of the United States would double by 1975. Disposable income was expected to increase by 54 per cent (41, p. 63). The increase in food production associated with an increase in gross product and disposable income cannot be answered by input-output analysis, since gross product, disposable income, and the final bill of goods are not identical concepts.

The results of input-output analysis could be used to examine the effects of changes in consumption expenditures if the administrative authority would make estimates of expected change in the final bill of goods included in the input-output analysis.

If the final bill of goods was estimated to increase by 50 per cent, what would agricultural production increase? Since input-output models assume linear homogeneous production functions of degree one, it must follow that a 50 per cent increase in the total final bill of goods would increase inputs by 50 per cent. In addition, each sector supplying the final bill of goods would all produce 50 percent more output. Individual commodities such as food commodities cannot be isolated out in the model used in this study.

The total increase in the agricultural sector is given simply by the underlying assumptions of the model; however, input-output analysis does indicate how this increased output must be distributed among the other sectors of the economy. Table 12 indicates the percentage of the increased net output in the two agricultural sectors that would flow to all sectors. Also, it indicates the quantity of agricultural production necessary for a 50 per cent increase in any one part of the final bill of goods.

Table 12. Thousands of Dollars of Additional Net Output from Agriculture for a 50 Per Cent Increase in the Final Bill of Goods

Sectors consuming the net output	Primary agricultural production	Secondary agricultural production	Percentage distribution	
			Primary agriculture	Secondary agriculture
Primary agricultural production	318,385	19,100	2.9	.2
Secondary agricultural production	606,765	906,635	5.6	10.2
Industry and services	8,947,500	7,433,995	82.1	83.7
Foreign trade	281,825	99,540	2.6	1.1
Government	743,020	421,810	6.8	4.8
Total	10,897,490	9,302,890	100.0	100.0

2. Changes in output

Agricultural policies of the past, as well as prospective ones, attempt to change agricultural output through acreage controls and marketing quotas. Resulting changes, therefore, result not from choice reflected by consumers but from attempted control of inputs. The effects of agricultural production control programs can have effects on farm income and on the flow of resources between the agricultural sector and other sectors of the economy. The farm income effects depend upon the demand elasticity for agricultural products; thus they cannot be analyzed here. The input-output procedure can be used, however, to reflect possible effects of increases and decreases in net output of agricultural production upon resource flows among the sectors of the economy.

The coefficients of the basic system of equations describe the relationships between the flows of resources among the sectors of the economy and specified levels of net output of a single sector. From Table 4, the following relationships for 1949 are derived:

1. Primary agricultural production:

$$x_{21} = .03200\ X_1$$

$$x_{31} = .33631\ X_1$$

$$x_{51} = .03522 \, X_1$$

$$x_{61} = .35601 \, X_1$$

2. Secondary agricultural production:

$$x_{12} = .65166 \, X_2$$

$$x_{32} = .14520 \, X_2$$

$$x_{52} = .00313 \, X_2$$

$$x_{62} = .28039 \, X_2$$

where x_{ij} is the flow of net output from the i-th sector to the j-th sector, and X_j is the net output of the j-th sector.

By assuming a change in the net output of the agricultural sector and determining the new output, the flow of resources from other sectors to the agricultural sector producing the new output can be determined from the above relationships. An increase of one dollar's worth of net output from primary agricultural production in 1949 would have necessitated .032 dollar's worth of net output from secondary agricultural production, .336 dollar's worth of net output from industry and services, and .035 dollar's worth of government services. An increase of one dollar's worth of net output from secondary agricultural production in 1949 would have necessitated .651 dollar's worth of net output from primary agricultural production, .145 dollar's worth of net output from industry and services, and .003 dollar's worth of government services.

More specifically, effects of a 10 per cent change in the net output of each sector upon the net output of all other sectors are given in Tables 13 and 14. A 10 per cent change in primary agricultural net output brought about by production control in 1949 would decrease secondary agricultural production by .392 per cent, industry and services net output by .339 per cent, and government services by .140 per cent. A 10 per cent change in secondary agricultural net output brought about by production control in 1949 would decrease primary agricultural production by 5.310 per cent, industry and services by .119 per cent, and government services by .010 per cent.

If the static conditions of the input-output model hold, a greater absolute change in agricultural output could be facilitated by 10 per cent control program in secondary agriculture than in primary agriculture. In Table 14, the absolute effect of a 10 per cent reduction in secondary agricultural output was a reduction in primary agricultural output by 1.1 billion dollars whereas the absolute effect of a 10 per cent reduction in primary agricultural output was a reduction in secondary output of only 70 million dollars. The effects of a 10 per cent reduction in secondary

Table 13. Per Cent of Increase in Net Outputs of Sectors Supplying Inputs Associated with a 10 Per Cent Increase in Net Outputs[1]

Sectors supplying inputs	Year	Sectors consuming the inputs in producing net output: Primary agricultural production	Secondary agricultural production	Industry and services	Foreign trade (exports)	Government
Primary agricultural production	1949	--	.05310	.03348	.00823	.00242
	1939	--	.05333	.02408	.00595	.01048
	1929	--	.05729	.02448	.01117	--
Secondary agricultural production	1949	.00392	--	.08616	.00022	--
	1939	.00666	--	.07965	.00012	--
	1929	.00659	--	.07966	.00025	--
Industry and services	1949	.00339	.00119	--	.00490	.00925
	1939	.00243	.00184	--	.00349	.00763
	1929	.00272	.00138	--	.00441	.00448
Foreign trade (imports)	1949	--	--	.04305	--	.03850
	1939	--	--	.06689	--	.00303
	1929	--	--	.06779	--	.00234
Government	1949	.00140	.00010	.05781	.00126	--
	1939	.00319	.00019	.06932	.00453	--
	1929	.00474	.00038	.06962	.00301	--

[1]Per cent calculated by dividing data in Table 14 by the corresponding net outputs (Tables 1, 2, and 3).

Table 14. Absolute Change in Net Output of Sectors Supplying Inputs and Labor Services Associated with a 10 Per Cent Change in Net Output (thousands of dollars)[1]

Sectors supplying inputs	Year	Sectors consuming the inputs in producing net output: Primary agricultural production	Secondary agricultural production	Industry and services	Foreign trade	Government
Primary agricultural production	1949	--	1,157,476	729,783	179,468	52,682
	1939	--	388,203	175,270	43,331	76,300
	1929	--	649,659	277,648	126,628	--
Secondary agricultural production	1949	69,744	--	1,530,405	3,972	--
	1939	37,334	--	446,434	669	--
	1929	44,074	--	533,200	1,675	--
Industry and services	1949	733,000	257,894	--	1,058,665	1,998,764
	1939	186,500	91,025	--	267,834	586,000
	1929	236,800	120,038	--	384,105	389,900
Foreign trade	1949	--	--	463,568	--	414,600
	1939	--	--	152,247	--	6,900
	1929	--	--	298,223	--	10,300

[1]Computed by multiplying 10 per cent of net outputs (Tables 1, 2, and 3) by the input coefficients (Tables 4, 5, and 6).

Table 14 (Cont'd)

Sectors supplying inputs	Year	Sectors consuming the inputs in producing net output				
		Primary agricultural production	Secondary agricultural production	Industry and services	Foreign trade	Government
Government	1949	76,766	5,555	3,162,824	69,300	--
	1939	42,574	2,577	924,249	60,400	--
	1929	59,316	4,766	871,318	37,700	--
Household (labor)	1949	775,924	498,032	11,730,900	--	2,042,400
	1939	254,206	159,065	3,912,200	--	762,900
	1929	412,113	229,402	4,485,100	--	435,600

agricultural production on industry flows was less as well as the effects on government revenues and the quantity of labor services. The greater absolute reduction in agricultural output through controls on secondary agriculture would be offset somewhat by the magnitude of the 10 per cent reduction in net output of the sector since primary agricultural net output slightly exceeds secondary agricultural net output (Table 1).

As previously indicated, estimates made by the United States President's Material Policy Commission (41, p. 65) indicated that primary agricultural production must increase by 30.3 per cent, and secondary agricultural production must increase by 37.9 per cent to supply the demand for food products in 1975. If one assumes these increases in the net output of the primary and secondary agricultural sectors, the effect of these increases upon the net outputs of sectors supplying resources to the agricultural sectors can be observed by input-output analysis. Table 13 gives the per cent increase in net output of sectors supplying resources to primary agriculture associated with a 10 per cent increase in primary agricultural production. A 30.3 per cent increase in primary agricultural production in 1949 would necessitate a 1.18 per cent increase in secondary agricultural net output, a 1.02 per cent increase in industry and services net output, and a .42 per cent increase in government services. A 37.9 per cent increase in secondary agricultural net output in 1949 would necessitate a 20.1 per cent increase in primary agricultural net output, a .45 per cent increase in industry and services net output, and a .03 per cent increase in government services.

The absolute flows of resources to the agricultural sector under the above assumed increases in net outputs are obtained from Table 14. A 30.3 per cent increase in primary agricultural net output in 1949 would necessitate 211,324 thousands of dollars' worth of resources from secondary agricultural production, 2,220,990 thousands of dollars' worth of resources from industry and services, 232,600 thousands of dollars' worth of government services, and 2,351,049 thousands of dollars' worth of labor services. A 37.9 per cent increase in secondary agricultural net output in 1949 would necessitate 4,386,834 thousands of dollars' worth of resources from primary agricultural production, 977,418 thousands of dollars' worth of resources from industry and services production, 21,053 thousands of dollars' worth of government services, and 1,887,541 thousands of dollars' worth of labor services.

Increases and decreases in net outputs of the agricultural sectors may come about by conditions other than those already mentioned. Emphasis on soil conservation and an increase in forage crops in the rotations on farms would lead to a reduction in the net output in primary agricultural output as a result of the shifting of acreage in cash crops to forages of lower value. These shifts may also result in increased net output if forages were complementary to grain in the rotations. The increase in forage in the primary agricultural sector often necessitates increased outputs of secondary agricultural production because of increased livestock production on farms where forage is an important part of a soil conserving program. Effects of soil conserving policies directed at affecting agricultural output can be analyzed once the magnitude of the shift in production has been estimated.

3. Changes in employment

So far the effects of changes in the final bill of goods as a result of changes in consumers' demand and changes in agricultural outputs as a result of agricultural policy have been the only considerations. The Leontief system also provides the information for an analysis of the effects of these changes upon employment. From Tables 4, 5, and 6, the following linear relationships between net output and employment can be constructed for 1949, 1939, and 1929, respectively:

1949	1939	1929
$x_{61} = .35601\ X_1$	$x_{61} = .34919\ X_1$	$x_{61} = .36342\ X_1$
$x_{62} = .28039\ X_2$	$x_{62} = .28378\ X_2$	$x_{62} = .34275\ X_2$
$x_{63} = .54268\ X_3$	$x_{63} = .50906\ X_3$	$x_{63} = .51520\ X_3$
$x_{65} = .37332\ X_5$	$x_{65} = .57215\ X_5$	$x_{65} = .34804\ X_5$.

These relationships determine the dollar value of employment required for any given level of net output. The new outputs may result from either changes in the final bill of goods or changes in output achieved through government programs. An increase of one dollar's worth of output in primary agricultural production required an increase of .356 dollar's worth of agricultural labor in 1949. Similar inferences can be made from the relationships existing in the other sectors of the economy.

Direct comparisons among all the coefficients are not possible since a dollar's worth of agricultural labor may not be the same as a dollar's worth of industrial labor or government labor. Also the difference between any pair of coefficients within a single year must allow for work preferences of individuals working in the two sectors. The common error made is to impute the difference to inefficient allocation of labor services in the economy. In this study labor services are measured in dollar value instead of man-hours. A difference in the coefficients of agriculture and industry can only be interpreted if something is known about the amount of physical exertion required per dollar of output in the two sectors and the wage rates. If we assume that the amount of physical exertion required per unit of output were the same in agriculture and industry, then the difference in the value of labor required per unit of output would reflect differences in the return to labor in the two sectors.

A direct comparison between the labor coefficients for primary and secondary agriculture can be made subject to slight reservation on the interpretation. The work preference factor is limited to personal preferences among enterprises within the agricultural economy. In many cases, individuals are involved in the production of both sectors. However, there are firms in agriculture which specialize in one or the other. The principle of opportunity costs, assuming constant labor coefficients, would indicate that the two coefficients should tend to be equated. The two are remarkably close considering the possible error in the data for approximating labor coefficients for agriculture. The coefficient tends to be larger for primary agricultural production than for secondary agricultural production, especially in 1939 and 1949. An increase of one dollar's worth of net output in primary agricultural production in

1949, necessitated .356 dollar's worth of agricultural labor, and a dollar's worth of secondary agricultural production necessitated .280 dollar's worth of agricultural labor. This means that a unit of labor would produce a greater value of product in secondary agriculture. Studies in production economics continually point out the possibilities for agricultural firms to increase income by increasing livestock production. Capital limitations restrict many farmers from expanding livestock organizations and thus they continue to employ their resources in crop production.

These conclusions with respect to labor productivity are not conclusive, but the results of this study are consistent with other findings. Other methods are more suitable for a study of the marginal productivity of agricultural resources within the firm.

The effects of changes in the final bill of goods upon the level of employment within a sector may also be observed from the Leontief system. By substituting the relationship between the final bill of goods and the net output of the sector into the appropriate labor relationship given above, the effect of a change in any part of the final bill of goods on employment can be determined. The 1949 relationships for agriculture would appear as follows:

$$x_{e1} = .35601\ (1.05925y_1 + .70528y_2 + .10186y_3 + .28372y_4 + .03912y_5) \quad (4.16)$$

$$x_{e2} = .28039\ (.06356y_1 + 1.05383y_2 + .08463y_3 + .10020y_4 + .03912y_5) \quad (4.17)$$

The equations may be simplified as follows:

$$x_{e1} = .3771y_1 + .2511y_2 + .0363y_3 + .1010y_4 + .0245y_5 \quad (4.18)$$

$$x_{e2} = .0178y_1 + .2955y_2 + .0237y_3 + .0281y_4 + .0110y_5 \quad (4.19)$$

G. Structural Change

Three different years, 1949, 1939, and 1929, were chosen for this investigation for the purpose of examining changes in the structural flow matrix over time. The linear relationships of production assumed in the model may limit the usefulness of the results in determining the effects of agricultural policy. Some knowledge as to the changes which occur in the structural flow matrix over time should aid in the interpretation of the results and establish bounds on the validity of the linear assumptions. Structural changes in the structural flow matrix could result from two phenomena in economic theory: 1. If the relationships are non-linear and the linear approximation in the model does not represent the true relationship, a movement on the non-linear production function of the sector will result in a different structural flow matrix at a new point in time. 2. With technological change or a shift in the production function, the structural flow matrix will be different even at the same level of resource use. Both are important in respect to using the Leontief system as a guide to policy decisions. Changing technology may necessitate more drastic control programs than would be indicated by input-output analysis in order to reduce net output to the desired level. Movement on a non-linear relationship approximated by a linear model of input-output analysis may cause one to under-estimate or over-estimate the expected results. In Figure 1 a technological change is illustrated by a shift in the production relationship from OA to OB. The input-output analysis based on the relationship OA would be inappropriate for determining the necessary reduction in output through control programs if OB were the actual relationship at the time the programs were put into effect. A movement along the linear relationship tangent to OA would result in an expected output OE from OX resources, but if the actual relationship is OA, the output would be only OD from OX resources.

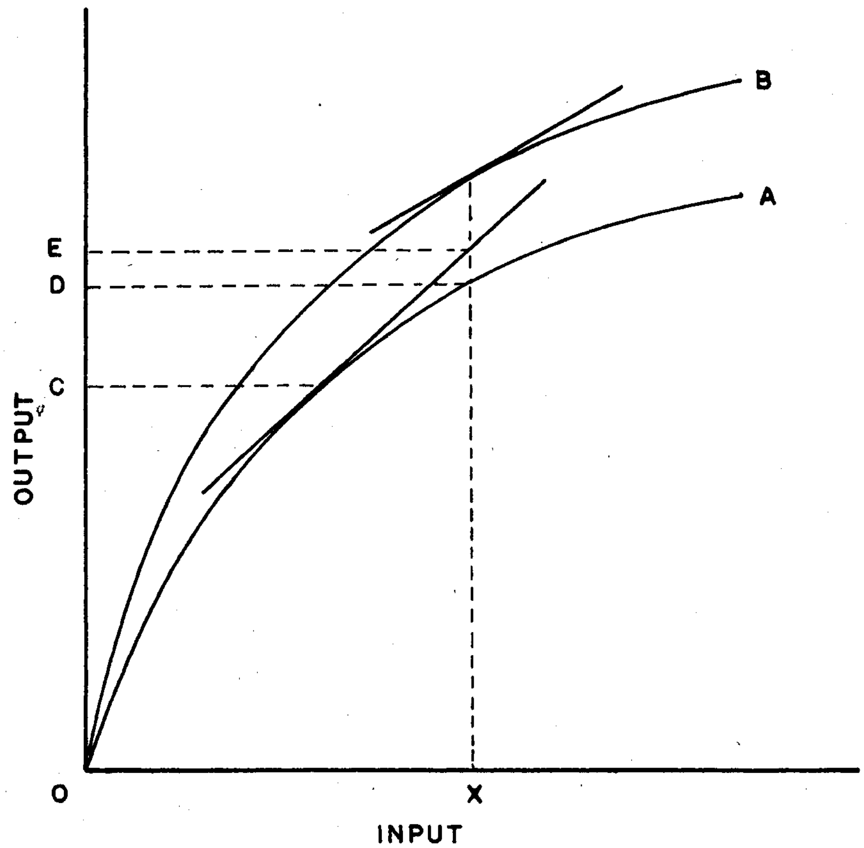

Figure 1. An illustration of technological change vs. movement on the production function

Changes in the structural flow matrix over time cannot always be related to one cause or the other. By examining the structural flow matrix, it may be possible to detect overall technical trends in the aggregate use of resources and output relations.

H. Adjusting the Data to a Base Price Level

To facilitate the analysis of structural change in the periods 1929 to 1939 and 1939 to 1949, the data must be reduced to a single price level. The data were converted to the 1939 price level.

To adjust the data in the input-output flow tables to a 1939 price level (Table 15), the following indices were used for each respective sector:

Sector	Index
Primary agricultural production	Index of prices received by farmers
Secondary agricultural production	Index of prices received by farmers
Industry and services	Wholesale price index
Foreign trade	Index of prices of foreign trade
Government	Index of consumers' prices

From the adjusted input-output flow tables, the technical coefficients of production were derived in the same manner as for Tables 4 and 6. The coefficients for 1939

Table 15. Distribution of Inputs and Outputs of the United States Economy for 1929, 1939, and 1949 Expressed in Terms of 1939 Dollars
(millions of dollars)

Sectors producing the output	Year	Sectors consuming the output (inputs): Primary agricultural production	Secondary agricultural production	Industry and services	Foreign trade (exports)	Government	Household consumption	Net outputs
Primary agricultural production	1929	--	3,946	1,686	769	--	486	6,887
	1939	--	3,882	1,752	433	763	448	7,279
	1949	--	3,889	2,452	603	177	202	7,324
Secondary agricultural production	1929	293	--	3,554	11	--	602	4,462
	1939	373	--	4,464	6	--	760	5,605
	1949	271	--	5,964	15	--	670	6,921
Industry and services	1929	1,919	972	--	3,113	3,160	61,395	70,561
	1939	1,865	910	--	2,678	5,860	65,537	76,850
	1949	3,641	1,281	--	5,259	9,929	87,274	107,385
Foreign trade (imports)	1929	--	--	1,863	--	64	821	2,749
	1939	--	--	1,522	--	69	684	2,276
	1949	--	--	2,039	--	1,824	874	4,738
Government	1929	481	38	7,070	305	--	2,259	10,156
	1939	425	25	9,242	604	--	3,036	13,334
	1949	447	32	18,418	403	--	12,558	31,860

remain the same. The input coefficients expressed in terms of 1939 dollars are given in Table 16.

To obtain the interdependence coefficients in terms of 1939 prices, it would ordinarily be necessary to find the inverse of the 1949 and 1929 structural flow matrices expressed in 1939 dollars. This would increase the cost of computations by two-thirds since all the computations must be repeated for two years. In order to avoid these additional costs, a transformation technique was used to derive the adjusted interdependence coefficients. The use of this procedure becomes more important as the model increases in number of equations. Thus a brief discussion of the transformation is appropriate for future research employing input-output analysis.

The transformation of the original matrix of interdependence coefficients is as follows:[1]

$$(A')^{-1} = R\ A^{-1}\ R^{-1} \qquad (4.20)$$

where

$(A')^{-1}$ is the matrix of adjusted interdependence coefficients.

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[1]The writer is indebted to Dr. Harry Goheen, Statistical Laboratory and Department of Mathematics, Iowa State College, for suggesting the transformation and deriving the procedure.

Table 16. Technical Input Coefficients and Consumption of the United States Economy for 1929, 1939, and 1949 Expressed in Terms of 1939 Dollars

Sectors producing the output	Year	Sectors consuming outputs (inputs)					
		Primary agricultural production	Secondary agricultural production	Industry and services	Foreign trade (exports)	Government	Household (consumption)
Primary agricultural production	1929	--	.88437	.02390	.33781	--	486,204
	1939	--	.69258	.02281	.19038	.05722	448,764
	1949	--	.56197	.02284	.16328	.00557	202,028
Secondary agricultural production	1929	.04266	--	.05038	.00491	--	602,361
	1939	.05128	--	.05809	.00294	--	760,813
	1949	.03711	--	.05554	.00419	--	670,547
Industry and services	1929	.27865	.21805	--	1.36735	.31116	61,395,642
	1939	.25619	.16239	--	1.17672	.43948	65,537,367
	1949	.49714	.18509	--	1.42379	.31165	87,274,453
Foreign trade (exports)	1929	--	--	.02642	--	.00634	821,331
	1939	--	--	.01981	--	.00517	684,633
	1949	--	--	.01899	--	.05728	874,130
Government	1929	.06985	.00867	.10020	.13436	--	2,259,959
	1939	.05848	.00460	.12027	.26537	--	3,036,000
	1949	.06103	.00467	.17152	.10926	--	3,693,791

R is a diagonal matrix of the ratios of the indices used to adjust the input-output flow tables.

A is the structural flow matrix.

The derivation of the procedure is as follows:

Let

A be the structural flow matrix.

X be the matrix of input-output flows with the net outputs in the diagonal.

P be a diagonal matrix of the reciprocals of the net outputs of the sectors.

R be a diagonal matrix of the ratios of the indices of the two years.

A' be the adjusted structural flow matrix.

S be a diagonal matrix of the reciprocals of the adjusted net outputs of the sectors.

The structural flow matrix is derived by:

$$A = XP \tag{4.21}$$

then

$$A^{-1} = P^{-1}X^{-1} \tag{4.22}$$

or

$$X^{-1} = PA^{-1} \ . \tag{4.23}$$

The adjusted structural flow matrix is derived by:

$$A' = RXS \tag{4.24}$$

then

$$(A')^{-1} = S^{-1}X^{-1}R^{-1} \tag{4.25}$$

substituting for X^{-1}:

$$(A')^{-1} = S^{-1}PA^{-1}R^{-1} \tag{4.26}$$

but

$$S^{-1}P = R \tag{4.27}$$

therefore

$$(A')^{-1} = RA^{-1}R^{-1} \; . \tag{4.28}$$

Diagonals of A^{-1} and $(A')^{-1}$ are identical after the transformation has been performed. Input-output studies indicate that the diagonal of the inverse of structural flow matrices approximate 1. The economic interpretation of the diagonal element is that a one dollar's worth of change in the direct demand for a product of the sector will increase or decrease the net output of the sector by one dollar's worth. The greatest limitation is that the procedure destroys the accuracy of the checks; however, when the adjusted relationships were used to predict the respective adjusted net outputs, the accuracy seemed satisfactory.

The results of the transformation of the interdependence coefficients are given in Table 17.

I. Technical Change Over Time

One method of observing technical change over time through input-output analysis is by a backward prediction of the net outputs (22, p. 153). The 1939 bill of goods and 1949 interdependence relationships expressed in 1939 dollars are used to predict the 1939 net outputs. The same procedure would be used with the 1929 bill of goods and the 1939 interdependence relationships. The discrepancy between the actual and the predicted net outputs indicates an existence of technical change.

Table 18 shows the results when these procedures were applied to the sectors in this study for the two periods 1929 to 1939 and 1939 to 1949.

In examining the predicted net outputs for 1929, it can be seen that all the net outputs are reasonably close to the actual net output except for secondary agricultural

Table 17. Interdependence Coefficients for 1929, 1939, and 1949 Expressed in 1939 Dollars

Sectors producing the outputs	Year	Sectors consuming the outputs: Primary agricultural production	Secondary agricultural production	Industry and services	Foreign trade	Govern- ment
Primary agricul- tural production	1929	1.06765	.96359	.08776	.40526	.02988
	1939	1.06679	.75341	.08686	.33430	.10095
	1949	1.05925	.66548	.07540	.23711	.04351
Secondary agricul- tural production	1929	.06361	1.06945	.05975	.09192	.01917
	1939	.07450	1.06311	.06989	.10898	.03554
	1949	.06736	1.05383	.06639	.08874	.02618
Industry and services	1929	.35777	.56133	1.10922	1.39769	.35400
	1939	.34039	.42089	1.12527	1.52864	.52192
	1949	.56493	.56178	1.14734	1.39158	.44068
Foreign trade	1929	.01016	.01568	.03007	1.03872	.01594
	1939	.00729	.00887	.02305	1.03276	.01589
	1949	.01978	.01869	.03348	1.04598	.07045
Government	1929	.11214	.13460	.12114	.28474	1.03950
	1939	.10560	.10192	.14685	.47796	1.07305
	1949	.15798	.14000	.20416	.34142	1.08414

Table 18. Prediction of the Adjusted Net Outputs for 1929 and 1939 with the Adjusted Interdependence Coefficients for 1939 and 1949 (thousands of dollars)

Sector	1939 net output	Predicted 1939 net output from 1939 bill of goods and 1949 adjusted coefficients	1929 adjusted net output	Predicted 1929 net output from 1929 adjusted bill of goods and 1939 coefficients
Primary agricultural production	7,279,815	6,217,511	6,887,985	6,808,042
Secondary agricultural production	5,605,189	5,323,253	4,462,328	5,137,367
Industry and services	76,850954	78,165,148	70,561,333	71,940,738
Foreign trade	2,276,098	3,147,284	2,276,945	2,308,205
Government	13,334,000	17,082,701	10,156,162	11,946,298

production. This is consistent with Leontief's findings when he employed this procedure on an analysis of industrial sectors of the economy over the same period. His conclusion was that little technical change was evidenced except in the railroad industry. From this study it might be concluded that little technical change occurred during the period except in secondary agricultural production. It is difficult to determine what the specific changes were in secondary agricultural production. In comparing the technical production coefficients for secondary agricultural production in Table 16, the input per unit of net output declined considerably for both primary agricultural and industry and services products from 1929 to 1939. In Table 15, net output of secondary agricultural production increased from 4.4 billion dollars to 5.6 billion dollars, whereas inputs from secondary agriculture and industry and services declined. A greater output from a smaller quantity of input is indicative of technological progress. This study is not specific enough to determine where this progress occurred in agricultural production.

When the backward prediction was applied to 1939 net outputs, the results were quite different than for the earlier ten year period. It might be concluded that technological change took place in all sectors except secondary agricultural production. The period from 1939 to 1949 included the war period which would be expected to bring about more rapid technological change than the previous period which included the depression years. Specific changes cannot be isolated, and the effects are net effects of all changes in technology.

A more objective method of observing the difference between the two estimates of net output obtained by the backward prediction technique would be desirable. As yet, the literature has not dealt with this problem.

J. Changes in Inputs and Outputs Over Time

It is interesting to analyze by inspection the technical input coefficients and the aggregate inputs of agriculture given in Tables 15 and 16.

In Table 16, column 1, a sizable change occurred in the input coefficient of industry and services products for primary agricultural production. This reflects the technological changes which have occurred in crop production over this period. The change from horse power to mechanical power would have increased this coefficient. Also the increased use of commercial fertilizer in crop production would have had the same effect. It is known that technological changes have taken place in these aspects of agricultural production. In Table 15, the quantity of inputs to primary agricultural production from industry and services measured in constant dollars almost doubled, whereas the net output of the sector increased only by 7 per cent. Acreage from which this output was produced might have actually decreased during this period, considering the probable increases in forage production and the effects of acreage control. The almost constant net output could have been maintained by the complementary relation between forage and grain production, improved varieties, or the apparent increase in the use of commercial fertilizers. These results are in no way conclusive, but from the viewpoint of the validity of the methodology, it does seem to reflect the technological changes which occurred in primary agricultural production during the period 1929 to 1949.

In Table 15, the net output of secondary agricultural production measured in constant dollars increased much more than primary agricultural output. Secondary net output increased over 30 per cent when comparing 1929 with 1949 outputs. Again this is indicative of technical changes in secondary agricultural production. The increase in net output of secondary agricultural production does not appear to have increased the value of the flow of resources from primary agricultural production because the dollar value remained almost constant. The input per unit of net output declined from .88 to .56 (Table 16). This could have been brought about by the use of cheap forages in livestock production which would place a greater volume of feed at the disposal of secondary production without any increase in the total value. This seems possible with the emphasis placed on reduction of acreage in soil depleting crops and increased forage production from soil conserving programs which took place during this same period. Another interesting phenomena seems evident in the inputs of industrial and services products. The input coefficient for products of industry and services remained relatively constant (Table 16), however, a significant change occurred in the total quantity of inputs. It might be deduced that the increase in secondary agricultural production did not come about by a technical change in feeding livestock commercial feeds. The change would appear to be a more extensive use of cheaper roughage feeds with approximately the same quantity of feeds supplementary to farm grown feeds. Again the input-output approach seems to reflect the technical changes which have taken place in secondary agricultural production over the past two decades.

All technical changes have not been confined to rotations and livestock rations. Other technical changes may have contributed to the size of the coefficients and quantities of resources used in production. Some technical changes in production technique may have compensating effects. For example, feeding operations may use more commercial feeds; however, innovations such as addition of antibiotics to rations may decrease the input coefficients. Only the aggregate effects of all technical change can be observed in the input-output analysis.

The foregoing empirical study has attempted to show the adaptability of the Leontief input-output analysis to agricultural production research. The evidence is meager and somewhat subjective. However, it does indicate that the results have some economic interpretation. The observations are historical, and the question still remains as to whether historical relationships are applicable to future agricultural policy. It cannot be over-emphasized that the analysis is static and subject to errors about which no knowledge is available; consequently, they have been ignored. Thus, the results and inferences are of a pilot nature which future research may easily refute. More realistic models, more accurate data, and interval estimation are all important considerations for further investigation.

The interest now turns to what use can be made of the linear Leontief system in dealing with problems arising in agricultural production. The final chapter will be devoted to this subject and will examine the validity of linear assumptions in agricultural production.

V. APPLICATION OF THE METHODOLOGY TO AGRICULTURAL RESEARCH

Two questions seem to be pertinent to the application of Leontief input-output analysis to agricultural research: (1) Are linear assumptions in production valid? (2) In what areas of agricultural research can the methodology be applied? This section will be devoted to a consideration of both questions based on the findings and experiences of conducting the present investigation.

A. Linear Relationships of Production

Assumptions of linearity of production are not new to agricultural production research. The explicit assumption of linear relationships (constant returns to scale) must be made in using the technique of imputing returns to factors of production. Generally market rates of return are imputed to all factors except to the one of interest, thus leaving the residual of the total product as a measure of the productivity of the final factor (13, p. 776). The traditional methods of analyzing farm business association records implicitly implied constant production coefficients. The budgeting technique employed by many agricultural economists in determining the optimum combination of farm resources is based on linear production relationships. This is not justification for adopting input-output analysis which also assumes linear relationships. Rather, an examination of the sectors included in the system should be made to determine the validity of linear assumptions. If it is found that linear relationships are not realistic, then it is important to recognize the limitations in using the linear approximations.

One hypothesis concerning agricultural input-output relations is that many aspects of agricultural production do follow linear relations. Particularly, where one is measuring aggregate net output of primary and secondary agricultural production, the assumption seems to have considerable validity. A single acre of corn production under a given technique could be duplicated in every fashion, and likewise, output would be duplicated. A single animal fed under a given technique could be duplicated by the same technical combination of inputs. Here the units of output are results of small individual production processes which do not follow linear relationships. That is, placing additional pounds on an animal does not come about by fixed input coefficients, but each 200 pound hog produced by the agricultural firm requires approximately the same combination of total inputs. When the units of output are the end products of a number of production processes, it would seem that the total output could be increased over a considerable range before the diseconomies of management, sanitation, etc., would become important and decrease returns.

It seems reasonable to believe that the quantities of many products included in net output of the agricultural sectors, particularly in secondary agricultural production, are affected more by the increase in the number of animals marketed or the number of acres produced rather than the level of output from each individual animal or acre.

Figure 2 illustrates the possibility of obtaining a given output from either of the two processes of production. PP and P'P' are iso-product curves representing two levels of output of pork production. An increase in the total output from PP to P'P' can be obtained either by producing more pounds on a given number of hogs or by marketing more hogs at the same weight. If the output PP is being produced with OA hogs and OC feed then by increasing the feed input to OE, the output P'P' would be achieved. This is the relationship facing the individual hog producer attempting to determine the most profitable weight to market a given number of hogs.

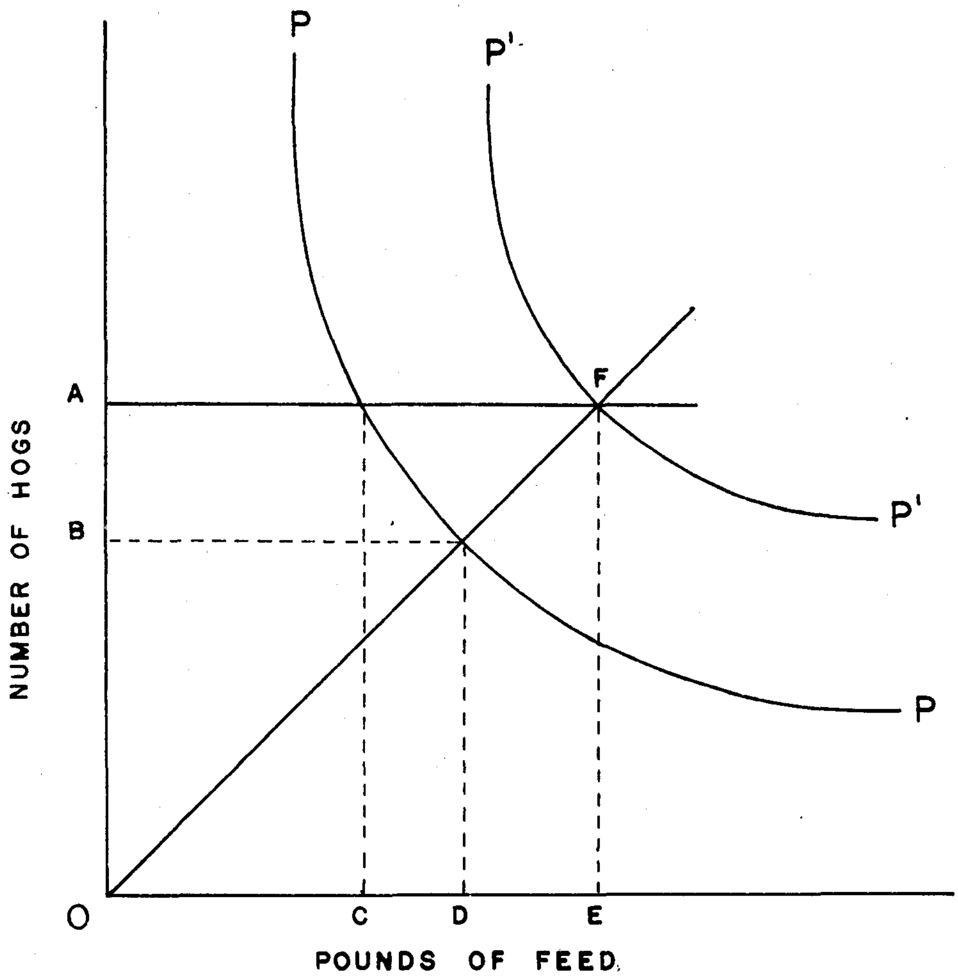

Figure 2. Illustration of combining resources to produce a given output

If the output PP is being produced with OB hogs and OD feed, then by increasing the number of hogs and the amount of feed in fixed proportions, the output P'P' would be achieved with OA hogs and OE feed. The individual hogs would now be marketed at the same weight as they were when the output PP was produced. This is the pattern of production when the hog producer varies the number of gilts farrowing and markets his hogs at approximately the same average weight.

Farmers have traditionally marketed hogs within the narrow weight range of 200 to 250 pounds, therefore, it might easily be found that aggregate changes in hog production are affected more by the number of hogs marketed than by heavier market weights of individual hogs. Outlook information would support this hypothesis since estimates of expected production of agriculture are based to a large extent on the number of pigs farrowed, number of cattle on feed, number of acres seeded, etc.

If aggregate outputs increase and decrease under the foregoing condition, it would appear that the production relationships of net output of agricultural products are linear homogeneous production functions of the first degree. Under these conditions, fixed coefficients of production in the input-output analysis would have considerable validity. Naturally, it would be impossible to believe that all aspects of production take place under fixed coefficients, but it would seem that linear assumptions are less serious on the aggregate level than on the level of individual firms.

One of the most serious limitations of input-output analysis is that, when changes in the level of net output in a sector is assumed, it must also be assumed that resources to produce the new output are available and can be drawn into the sector. This is unrealistic in agriculture where the quantity of land is fixed, and capital is not always available to an individual producer. The methodology assumes that when output in primary agriculture increases, land would vary as well as other resources. More acres of one crop generally means fewer acres of some other crop. If acres of land are not available for more production, then the only method of increasing production is to apply more resources to the acres already in production. Thus, relationships cannot be assumed linear. When additional capital is not available, other resources are combined with the fixed quantity of capital to increase agricultural production. Under these circumstances, constant input coefficients are not valid.

A limitation which is related to the linear relationships in the Leontief system arises when the relationships of one year are used to predict the changes which will occur under new agricultural policies. This study indicates that the constant coefficients do not hold over long periods of time probably because of technological change. Technological change is expected to continue for quite some time since many practices now known have yet to be adopted by agricultural firms. Any use of the input-output analysis to analyze the effects of new agricultural policies would necessarily be limited to very short periods of time and, even then, may be subject to considerable error. A continuous input-output study of the American economy might be useful in policy decision making and may provide information for the development of more realistic dynamic models.

B. Regional Input-Output Analysis

One promising area of agricultural research where the Leontief input-output analysis should prove to be appropriate is in the analysis of interdependence of regional agricultural production. This problem is of interest from both the standpoint of resource allocation and of effects of agricultural policy. Some pilot work in this field has been done by Isard (16). His investigations were concerned primarily with obtaining a suitable classification of industries into national, non-national, and local industries. The work is useful as a guide to developing an input-output model for regional analysis of agriculture. The essential revision in the model is that a system of equations are set up for the balance of regional production and consumption (23, p. 97). It is beyond this study to outline a mathematical model appropriate for studying regional relationships in agricultural production, but it appears that such an analysis could be applied with much less difficulty than that involved in a regional study of industries. Regional agricultural production appears to be "national" industries and thus could be added to the over all system with an equation for each regional area of agriculture. The most difficult problem is to obtain the estimates of the flows of output among the agricultural regions and the flows from the other sectors of the economy to each agricultural region. If such data could be obtained, the mathematical model would not be altered except that a larger system of equations would be involved.

VI. SUMMARY AND CONCLUSIONS

A study of agriculture as an industry is of economic importance for determining an efficient allocation of resources between sectors as well as within sectors of the economy and determining the effects of agricultural policies directed at controlling resource flows and net outputs of the economy.

Wassily W. Leontief of Harvard University introduced a mathematical technique commonly known as "input-output analysis" in which the economy is described by a system of linear equations establishing a balance among net outputs, flows of net output, and final consumption in the economy.

The system appears as follows:

$$\sum_{j=1}^{n} x_{ij} + y_i = X_i \qquad i \neq j \; (i = 1,2,\text{---}n) \qquad (6.1)$$

where n is the number of interrelated sectors of the economy; x_{ij} is the flow of net output from the i-th sector to the j-th sector; y_i is the direct contribution of the i-th sector to the final bill of goods; and X_i is the net output of the i-th sector.

The technical production coefficients are assumed constant, and, thus the production relationships for all sectors and all resources are:

$$x_{ij} = a_{ij}X_j \, . \qquad (6.2)$$

By substitution, the basic system of equations appears as follows:

$$X - \sum_{j=1}^{n} a_{ij}X_j = y_i \qquad (i = 1,2,\text{---}n) \, . \qquad (6.3)$$

By solving this system of equations the interdependence between the final bill of goods and net outputs is established.

$$\sum_{j=1}^{n} A_{ij}y_j = X_i \qquad (i = 1,2,\text{---}n) \, . \qquad (6.4)$$

The Leontief input-output analysis was used to study the interdependence of agriculture and other sectors of the economy in 1949, 1939, and 1929. Results of this study show the effect of changes in the final bill of goods to be produced upon the net output of the sectors included in the study and the level of employment. Effects of a change in the net output of a sector upon the net output of other sectors and the level of employment associated with any specified level of net output were also observed for the three years included in the study.

Not only was this study directed at an analysis of the United States economy but also at an introduction of the Leontief technique as a tool for analyzing problems of agricultural policy affecting production.

1. Primary agricultural production, defined as the output of agriculture derived from the culture of plant life, is highly interrelated with secondary agricultural production and industry and services production. A one dollar's worth of increase in the direct demand for primary agricultural production in 1949 required a .064 and .418 dollar's worth of secondary agricultural production and industry and services production, respectively.

2. Secondary agricultural production, defined as livestock production and storage activities, was most interrelated with primary agricultural production and industrial and services production. A one dollar's worth of increase in the direct demand for secondary agricultural production in 1949 required a .705 and .441 dollar's worth of increase in primary agricultural production and industry and services output, respectively.

3. This study would indicate that industry and services production of the United States economy is not highly

dependent upon agricultural production for producing its aggregate net output. Changes in the direct demand for industry and services output have little effect upon agricultural output. It must be emphasized that aggregate effects are observed and not effects of increase in demand for specific groups of commodities. Since derived demand for agricultural products is probably more important than the direct demand for agricultural products in the Leontief system, the interdependence of industry and services and the agricultural sectors indicates the effects of changes in derived demand for agricultural products. In 1949, a one dollar's worth of increase in the direct demand for industry and services products necessitated .102 and .085 dollar's worth of increase in primary and secondary agricultural production, respectively.

4. The results of the analysis of the foreign trade sector indicated the relationship between the final bill of goods and the flow of exports. Foreign trade analysis in this study was not appropriate for examining the effect of foreign trade policies, such as tariff changes and trade agreements.

5. Government is more interrelated with the industrial sector of the economy than either of the agricultural sectors

included in this study. Payments to government (inputs) from the industry and services sector exceeded the combined payments to government from both agricultural sectors.

6. The Leontief system is an appropriate tool for studying the aggregate interdependence of sectors of the economy. Aggregate effects of changes in demand for agricultural products and resource requirements associated with a given level of output can be observed and are important to policy decisions for agriculture. It must be emphasized that effects observed by the input-output technique are based on a static analysis, subject to errors of observation, based on constant production coefficients, and based on the assumption that techniques of production remain unchanged.

7. Results of this study are not adequate to determine the validity of the assumptions of input-output analysis. This study would indicate that the constant coefficients of input-output analysis are applicable for only short periods of time. Input-output analysis of the economy for a number of years can furnish information on the changes in the structural flow matrix which are caused by technological change or changing price relationships. This information would assist the scientist in predicting results of policies and determining policies to achieve ends in view.

8. Input-output analysis can be applied to other problems in research. Regional analysis of agriculture seems to be an appropriate area for the application of the input-output technique.

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IX. APPENDIX

A. The Source of the Data

The available data for an input-output analysis are at best rather unsatisfactory. Several of the variables are estimated from several sources of information, and in many instances it is necessary to rely heavily upon the knowledge of the subject matter in deriving the estimates. Census statistics are not available for all years, and the complete censuses of all sectors of the United States economy are not made in a single year. The Census of Manufactures was made in 1947 and will not be taken again until 1953. The Census of Business was last taken in 1948. The Census of Agriculture and the Census of Population are made each ten years and were last taken in 1950. The statistics from these censuses are generally not available until two or three years after the census year. For example, census years were selected for this study; however, the 1949 agricultural business obtained in the 1950 census was still not available for this study. Much of the data used in this study are based on an extrapolation of the census data and are later adjusted as new sources of information become available. This is the general practice used by the Department of Commerce in estimating national income statistics. In general, the data used in this study are estimates made and published by the Bureau of Agricultural Economics, the Department of Agriculture, the Department of Commerce, the Division of Labor Statistics, and the Bureau of the Census.

Little or no information is available concerning the reliability of the aggregate estimates of production and consumption in the United States economy. Tintner (28, p. 63) points out that this is a serious problem which needs considerable attention. Much could be done to improve input-output analysis if the agencies that prepare estimates of the aggregates needed for the analysis would also report measures of precision to be attached to these quantities.

B. Estimating the Variables x_{ij}, y_i, and X_i

The statistical model was developed in Section II, and a general description of the economic variables in the model was included in Section III. The detailed source of the data and the methods employed in deriving estimates of the variables where direct sources were not available will be discussed in this section. Limitations of these data in providing information on the flows of resources among the various sectors of the United States economy will be included when appropriate. Occasionally the source of information for estimating the variable was available for only one or two of the years studied. In such cases, it was necessary to use more than one source or devise a technique for estimating the data for the other years. All the relevant sources and procedures used in estimating a variable are included under the discussion of each variable.

1. Primary agricultural production

x_{12} The physical quantity of feed fed to livestock was obtained from the distribution of feed crops and other crops fed to livestock (34, pp. 7-10 and pp. 31-32) (38). Where the distributions of crops were estimated for the crop year rather than the calendar year, a moving average of two years each having a weight, .50, was used to estimate the quantity of feed fed to livestock during a calendar year. The quantities of each feed fed to livestock were multiplied by the average prices received by farmers during the calendar year. Average prices were obtained from (31, pp. 72-75), (32, pp. 72-75), and (39, pp. 499-503). Where monthly average prices were not given for feeds consumed by livestock, the seasonal average price reported in (38) was used.

Little information was available on the estimates of the value of pasture consumed by livestock. Jennings (18, p. 14) made estimates of the total feed units of pasture produced in the periods 1941-42, 1938-40, and 1929-33. These estimates were extrapolated to the years included in this study by the ratio of total acres in pasture to total acres in farms determined from the agricultural census (34). One pasture feed unit was equivalent to one pound of corn, thus the number of pasture feed units was multiplied by the average price of a pound of corn to determine the value of pasture consumed by livestock. A somewhat more refined method of estimating pasture value was devised by Jarrett (17, p. 79). His method gave an additional weight to pasture and range conditions to include consideration for weather.

The net increase in the value of stocks of grain and other crops stored on farms and in bins owned or controlled by the Commodity Credit Corporation was estimated by the difference between stocks of grain at the beginning and end of the year multiplied by its average price or seasonal average price (34) (38).

x_{13} The quantity of crops consumed by industry was obtained as a residual quantity. The residual quantity of the total production of the crop plus the change in inventories held on farms and in government bins after subtracting the disposition to feed, seed, direct consumption on farms, exports, and military procurement[1] was allocated to the industry and services sector. This residual was valued at the average price or seasonal average price depending upon the crop. This procedure was followed for all crops reported in (38) and the value of the residuals were added to obtain an estimate of the value of crop production flowing to the industry and services sector.

The value of forest products including value of free use timber and products of farm forestry (38) was added to the value of the residual of crop production discussed above.

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[1]The method of estimating the quantity of feed, direct consumption, and military procurement are discussed on pages 113, 117, and 118, respectively.

Exports are reported for the period July 1 to June 30 in (38), therefore a moving average of two years with weights, 1:1, was used to estimate the quantity of the crop exported during the calendar year.

The estimate of the quantity of the crop used for seed was the previous year estimate of disposition of the crop for seed. The previous year estimate of seed is based on the number of acres seeded the following crop year (38).

x_{14} The quantity of each crop exported was given in (38); however, these quantities were used only in determining the residual quantity flowing to the industry and services sector. The annual value of crops exported was obtained from foreign trade statistics (36). Using foreign trade statistics, avoided the use of a moving average in determining the annual export of crops. There are some discrepancies in the valuation of the flow from agriculture to exports since foreign trade statistics do not value the products at producer's prices which were used for all other flows from primary agricultural production. These discrepancies seem less serious in a small model since it seems reasonable to assume that these discrepancies fall within the error of the aggregate estimates. Other studies have given much consideration to this problem (6, p. 102).

The value of the following exported commodities were included in this variable:

grains.
hay.
fruits and preparations.
fresh fruits.
fresh vegetables.
nuts.
seeds.
tobacco, unmanufactured.
cotton, unmanufactured.
wood, unmanufactured.

x_{15} Crop production flowing directly to government includes those quantities procured for military exports and for military food use. Wheat was the only commodity which entered into this category in 1949. No quantities were given for the other two years included in the study. The quantity of wheat purchased by government under military procurement (38) was valued at the average price of wheat during the year.

This variable also includes the value of government payments under rental and benefit, soil conservation, price adjustment, price parity, Sugar Act, and production programs. The aggregate estimates were given in (38) for all years from 1929 to 1950. Including government subsidy payments to agriculture in this variable assumes that government subsidies to agriculture were an additional return to the output of agriculture above that which was received through the market.

y_1 The value of household consumption on the farm and the value of crops exchanged for consumption goods was obtained from the distribution of each crop (38). For those crop distributions reported on the basis of the crop year, a moving average of two years with equal weight, .50, was used to estimate direct farm household consumption during the calendar year. The total direct household consumption of farm crops was checked against the aggregate estimates of farm household consumption given in (38) under the estimate of gross farm income.

X_1 The net output of primary agricultural production is the sum of the dollar value of all the other variables included in the equation.

2. <u>Secondary agricultural production</u>

x_{21} The value of manure produced by livestock and assumed to be utilized in crop production was estimated on the basis of total value of feed consumed by livestock. The total value of livestock feed consumption determined for x_{12} in the equation for primary agricultural production was converted to corn equivalent by dividing the value of feed by the price of a ton of corn, using the average price of corn for the year. French (7, p. 69) estimated that a ton of corn produces 1.7 tons of manure. The tons of corn equivalent feed consumed by livestock was multiplied by 1.7 to obtain the estimate of tons of manure produced by livestock. The value of a ton of manure was estimated from the average NPK content of manure and the price of three mixed commercial fertilizers. An average ton of barnyard manure contains 10 pounds of nitrogen, 5 pounds of phosphorous, and 10 pounds of potassium (27, p. 209). The 1949 price of a pound of each nutrient was estimated to be .1156 for nitrogen, .0956 for phosphorous, and .0893 for potassium using the prices of three mixed fertilizers: 2-12-6; 3-12-6; and 4-12-4 (38). Prices paid by farmers for mixed fertilizers were not available for 1939 and 1929, therefore the value of a ton of manure for 1949 was adjusted by the index of prices paid by farmers for fertilizer (38). The estimated values of a ton of manure for the three years were \$2.53 for 1949, \$1.70 for 1939, and \$2.19 for 1929.

x_{23} The value of livestock production consumed by the industry and services sector was estimated by the total value of each class of livestock sold for slaughter plus the value of livestock products sold. Livestock products sold include dairy products, wool, mohair, and eggs. These estimates were derived from (38).

x_{24} Livestock and livestock products exported may not actually be considered as flowing directly from agriculture to the export market. In order to observe the interdependence between secondary agricultural production and exports, it was assumed that those products flowing from secondary agricultural production to the export markets did not go through any major change in form because of processing and handling for shipment to foreign markets. The value of the following commodities exported in the three years, 1949, 1939, and 1929 were included in the estimate of the variable (36):

animals, live, edible.
animals, live, inedible.
eggs, in shell.
wool, mohair, angora rabbit hair, unmanufactured.

y_2 Farm household consumption of livestock was obtained from (38) for each of the following classes of livestock: cattle, hogs, sheep and lambs, chickens, and turkeys. To this was added the value of milk products and eggs consumed by farm households (38).

X_2 The net output of secondary agricultural production is the sum of the dollar value of all other variables in the equation.

3. <u>Industry and services</u>

x_{31} The value of fertilizer and lime used on farms was obtained from (38) except for the year 1949. The 1949 estimates were considered to be subject to considerable adjustment; therefore, more recent estimates given in (29) were used for 1949.

The estimate of production expense for operation of motor vehicles includes expenditures for gasoline, oil, tires, and repairs on tractors. Forty per cent of the automobile expense is included in estimates for 1939 and 1929, and 50 per cent of the automobile expense is included in the 1949 estimate (38).

Depreciation on buildings, machinery and equipment reported in (38) was the amount needed to maintain the condition of farm buildings, machinery, and equipment at the beginning of the year in a constant state of repair. Only depreciation on machinery and equipment was included in this variable. The 1949 figure for total depreciation given in (38) was reduced by the value of new buildings, building repairs, and fence construction given in (29, p. 39). The 1939 and 1929 totals given in (38) were reduced by the expenditures and depreciation on buildings given in (33, p. 42).

Miscellaneous production expenses for insecticides, twine, ginning, irrigation, seed and nursery stock given in (38) are aggregated with electricity, insurance, veterinary services, dairy supplies, and other livestock expenses. Only estimates for seed expenses (30, p. 8) are given separately. Since seed includes the major part of the miscellaneous production costs of crop production, only this item was included in this variable. The balance of miscellaneous production expenses was allocated to secondary agricultural production.

x_{32} Depreciation and repair on buildings and fences were available for 1949 (29, p. 39). Estimates of these expenses for 1939 and 1929 included only depreciation and expenses on buildings (33, p. 42).

Miscellaneous production expenses less the value of seed purchased was included in this variable as was discussed under the variable x_{31}.

The value of commercial feeds purchased by the secondary agricultural production sector was estimated by multiplying the annual disappearance of feedstuffs by the average price per ton, bagged in wholesale lots, at leading markets (38). Disappearance of commercial feeds was given for the year beginning October 1; therefore, a moving average of two years with weights, .50, was used to estimate the disappearance during the calendar year. No comparable data are available for 1929. (38) gives the total production expense for feed bought for all years. The average ratio of total value of feed bought and value of commercial feed consumed in 1949 and 1939 was used to estimate the value of commercial feed bought by farmers in 1929.

x_{34} Exports of industrial production was obtained from foreign trade statistics (36). The total value of general exports was adjusted by the value of agricultural exports of crops and livestock included in x_{14} and x_{24} and the value of government sales abroad. (40, p. 155)

x_{35} Government purchases of goods and services from business (40, p. 155) was adjusted by the value of agricultural crops purchased by government included in x_{15}.

The balance on government subsidies and government enterprises found in the Department of Commerce's accounts of national income was included in this variable since in the model government enterprises were treated as a part of the industry and services sector of the economy (40, p. 155).

y_3 Consumption of industrial production and services by households was estimated from personal expenditures for durable and non-durable goods and services reported in national income statistics (40, pp. 198-199). The total personal expenditure reported was adjusted by deducting the value of food produced and consumed on farms (40, pp. 192-193), the value of fuel and ice produced and consumed on farms, and rental value of farm houses (40, p. 192-193).

X_3 The net output of industry and services was obtained by adding the dollar value of all other variables in the equation.

4. Foreign trade

x_{43} Imports consumed by industry and services were the total value of general imports less the value of imports allocated to households and government. The total value of general imports was obtained from (36).

x_{45} Gross purchases from abroad by government was given in the Department of Commerce's accounts of government expenditures in estimating national income (40, p. 155).

y_4 Data were not available on the flow of imports to the individual sectors of the economy. Leontief allocated imports to the sectors which produced similar products in the domestic economy. His procedure eliminates a great deal of judgment necessary in allocating imports to the sectors which use the imports. In this study imports were allocated by using the Department of Commerce's aggregation of general imports by economic classes: crude materials, crude foodstuffs, manufactured foodstuffs, semi-manufactures, and finished manufactures (36, p, 846). Manufactured foodstuffs and finished manufactures less government purchases from abroad were allocated to direct household consumption.

X_4 The net output of foreign trade was obtained by adding the dollar value of all other variables in the equation.

5. Government

The government equation shows the source of government receipts from other sectors of the economy including payments from households to government. To facilitate the allocation of government receipts, it was assumed that contributions to social insurance by employees were a payment to households for labor services. They were excluded from the government equation and included in payments to households in the form of supplements to wages and salaries.

x_{51} Real estate tax and personal property tax paid by agriculture were obtained from (38). All real estate tax paid by farm owners was included as a payment to government by the primary agricultural sector. The portion of personal property tax paid to government by the primary agricultural sector was estimated by the ratio of the total value of all livestock on farms at the beginning of the year and the total farm investment in livestock, machinery, and equipment at the beginning of the year.

x_{52} The share of personal property tax paid by agriculture not included in x_{51} constituted the estimate of personal property tax paid by secondary agricultural production.

x_{53} The estimate of corporate profits tax accruals was obtained from (40, p. 154). Indirect business tax and non-tax accruals were reduced by the total real estate and personal property tax paid by agriculture and included in this variable as the indirect business tax and non-tax accruals paid by the industry and services sector.

To balance the addition of government subsidy payments to agriculture in x_{15}, the total subsidy payment to agriculture was entered as a payment from industry and services sector to government. In the estimate of national income, the receipts of government from government enterprises and payments of subsidies by government are reported as a net subsidies less current surplus of government enterprises in the government expenditure account (40, p. 155). Since it was desirable to include the subsidy payment to the agricultural sector, it was assumed that these funds were also an income from surplus of government enterprises. This increased the net output of government reported in (40) (government expenditures) by the value of subsidies paid to agriculture.

x_{54} This variable was estimated by government revenues from customs and gross sales of government abroad (40, p. 154-155).

y_5 Personal tax and non-tax revenues not chargeable to business (40, p. 154) and contributions of employees to social insurance (40, p. 155) were used to estimate the payments of households to government. This represents the value of government services consumed by households.

X_5 Total government expenditures reported in (40) plus the subsidy payment to agriculture constitutes the value of net output of government.

6. Households (labor)

The data used to estimate the input of labor services in each sector of the economy except agriculture were obtained from the Department of Commerce estimates of national income (40). The estimate of wages and salaries for agriculture in national income statistics includes only hired labor and ignores the importance of family labor consumed in the agricultural sectors. Other sources are available for estimating the hours of labor required in agricultural production which do include family labor.

x_{61} An aggregate estimate of labor required by enterprises on farms in terms of man-hours was available in (38). The estimates were given for all livestock production, all crop production, and farm maintenance. The man-hours of labor required on farms for livestock production plus a proportionate share of the man-hours required for farm maintenance was multiplied by the average hourly wage for farm labor without board (38). The hourly wage was not reported for 1929, therefore, the wage per day without board was used and an eight hour day was assumed. The proportionate share of the labor requirement for farm maintenance was estimated by the ratio of all other inputs to primary agricultural production and the total of all other inputs to agriculture other than labor. The per cent allocated to primary agricultural production was 67.3 per cent for 1949, 68 per cent for 1939, and 73 per cent for 1929. The hourly wage rates were .68 dollars per hour for 1949, .20 dollars per hour for 1939, and .28 dollars per hour for 1929.

Wages and salaries paid forestry workers (40, pp. 160-161) and supplements to wages and salaries of forestry workers (40, pp. 162-163) were added to the value of farm labor used in primary agricultural production in deriving the estimate of the variable.

x_{62} The estimate of man-hours of labor required for crop production plus a proportionate share of the man-hours of labor required for farm maintenance (38) was multiplied by the same hourly wage rates used in estimating x_{61}. The proportionate share of man-hours of labor required for farm maintenance was estimated by the ratio of the total value of all other inputs to secondary agricultural production and the total value of all inputs to agriculture other than labor. The per cent allocated to secondary agricultural production was 32.7 per cent for 1949, 32 per cent for 1939, and 27 per cent for 1929.

x_{63} The wage bill for industry, services, and government enterprises was determined by adding the value of wages and salaries by industry given in (40, pp. 160-161) for all industries and services except farms, forestry, and general government. To this was added the value of supplements to wages and salaries for all industries and services except farms, forestry, and general government (40, pp. 162-163).

x_{65} Government salaries and wages were obtained from government expenditures (40, p. 155) by subtracting wages and salaries of government enterprises (40, p. 160-161) and adding supplements to wages and salaries of general government (40, pp. 162-163).